Hierarchical Riemannian Models for Pointset Shape Representations: Applications in Hypothesis Testing, Object Segmentation and Shape Clustering

PhD Defense Presentation
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by
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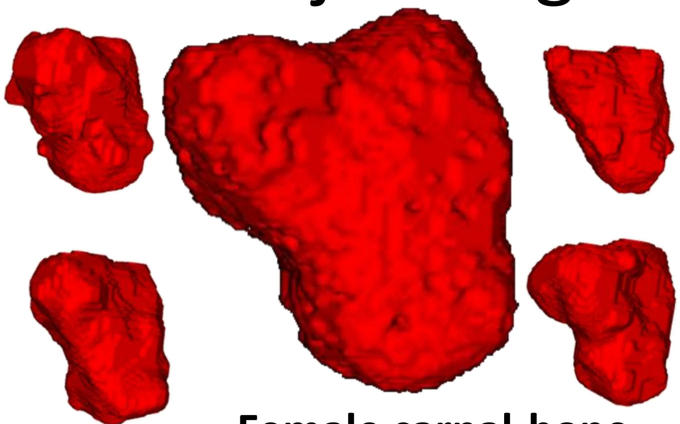
Under guidance of
Prof. Suyash P. Awate

Outline of the Talk

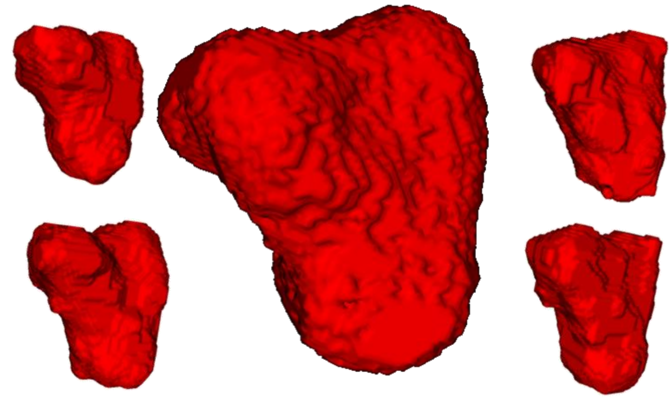
- Driving applications
  - Hypothesis testing, segmentation, clustering
- Statistical modeling on shapes
  - Formulation
- Multigroup hypothesis testing
  - Formulation, empirical evaluation
- Shape priors for object segmentation
  - Formulation, empirical evaluation
- Clustering a set of shapes
  - Formulation, empirical evaluation
- Conclusion

Application 1 – Hypothesis Testing

- Data: Object segmentations in multiple cohorts



Female carpal-bone examples

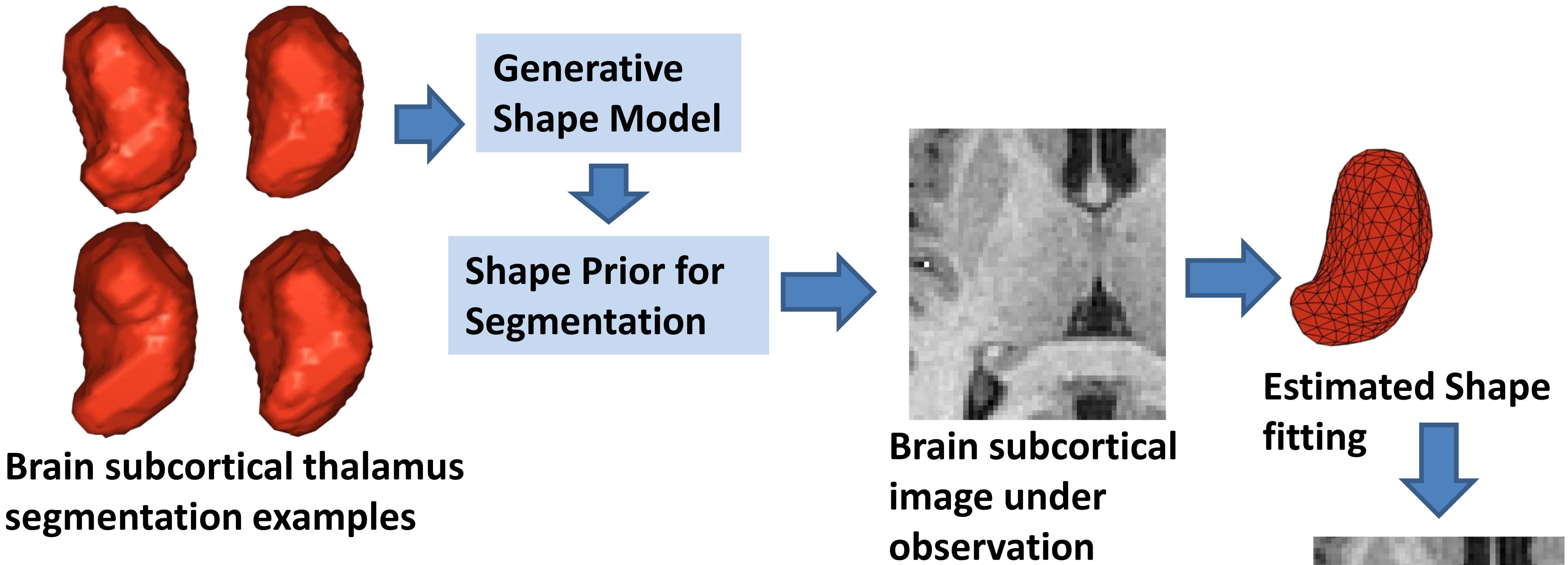


Male carpal-bone examples

- Task: Test null hypothesis that two cohorts have no difference in shapes
  - Need to learn a generative model of shape for each cohort / group
  - Visualizing group mean, modes of variation is key to clinical applications

Application 2 – Object Segmentation

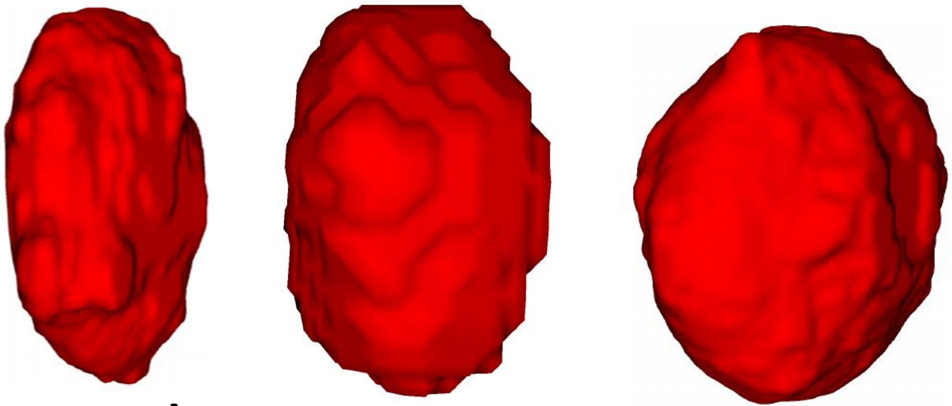
- Data: Object segmentations in a group of subjects; test image to be segmented



- Task:
  - Need to learn a generative model of shape
  - Generate segmentation using shape prior

Application 3 – Clustering

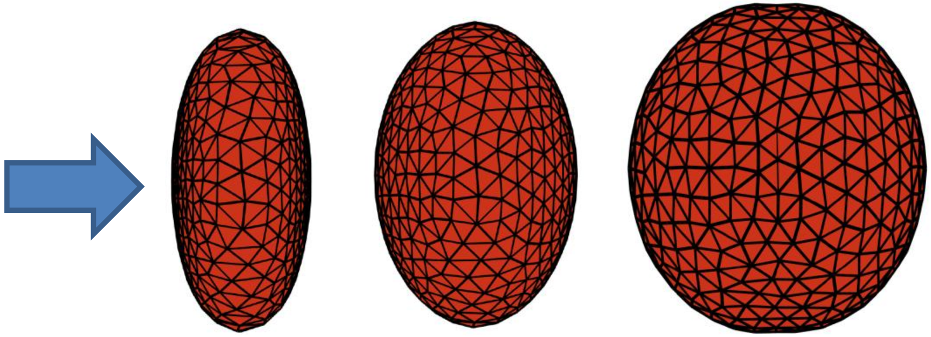
- Data: Set of object segmentations



Some examples

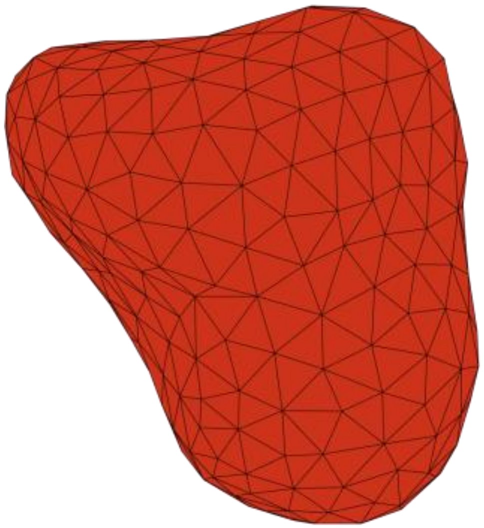
- Task: Cluster object shapes
  - Needs a generative model of shapes for each group
    - Estimate mean, modes of variation, for each cluster
  - Visualizing cluster mean, modes of variation is key to clinical applications

Estimated means for each cluster



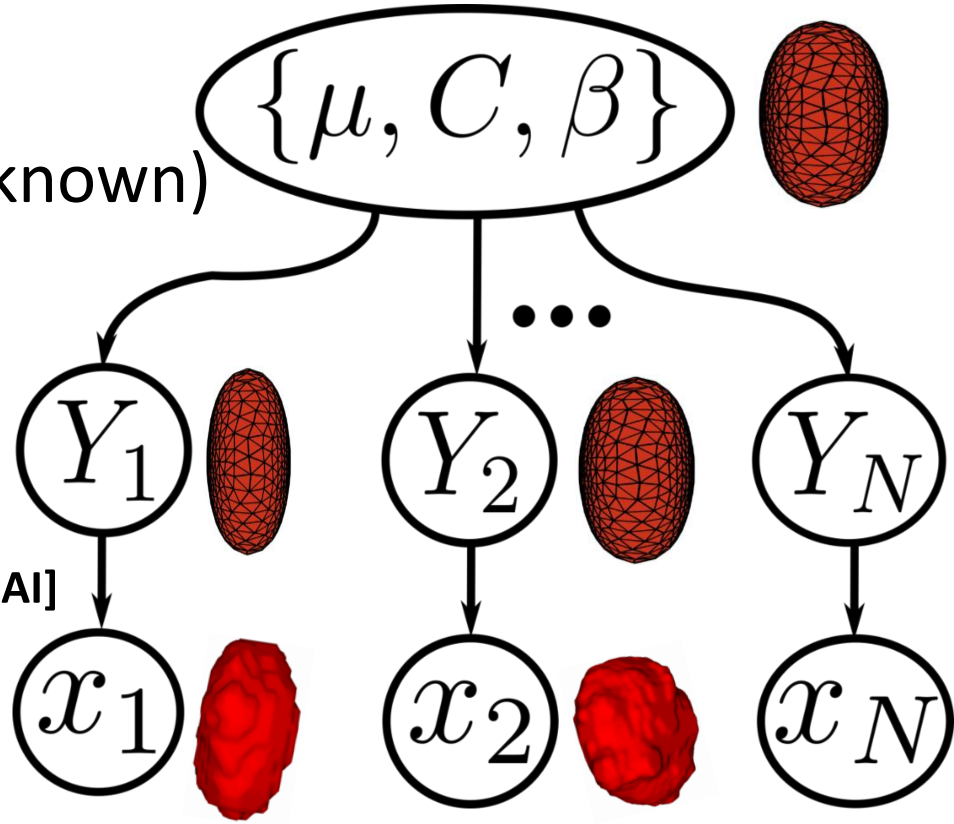
Representation of Shapes

- What is shape ?
  - Object shape is all the geometrical information that remains when location, scale, and rotational effects are filtered out from an object
  - We leverage the notion of **Kendall Shape space** that has a **non-Euclidean** structure
- Our representation of shape
  - Finite number of points on object boundary with triangular mesh



Statistical Model on Shapes

- Group
  - Data $x := \{x_i\}_{i=1}^{N}$
  - Individual shapes $Y := \{Y_i \in \mathbb{R}^{3J}\}_{i=1}^{N}$ (latent / hidden random variable)
  - Mean μ (unknown)
  - Modes of variation C (unknown)
  - Smoothness prior β (free parameter)



[A Gaikwad, SJ Shigwan, SP Awate, 2015 MICCAI]

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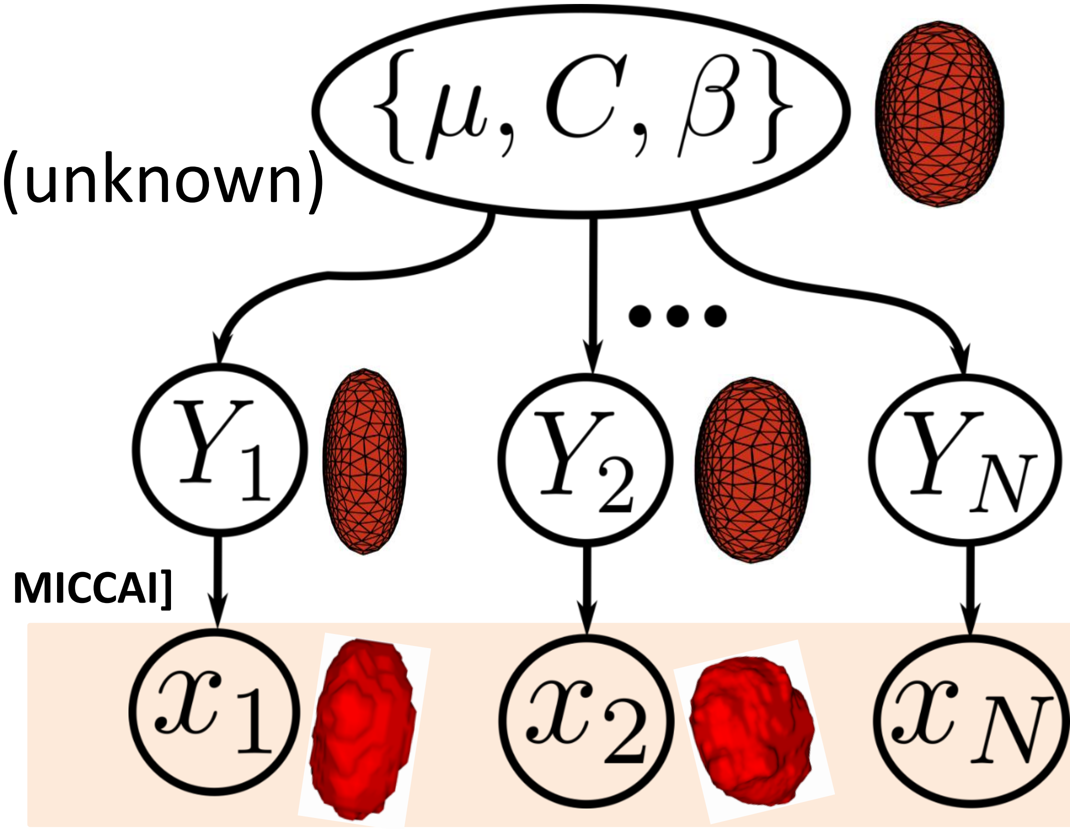

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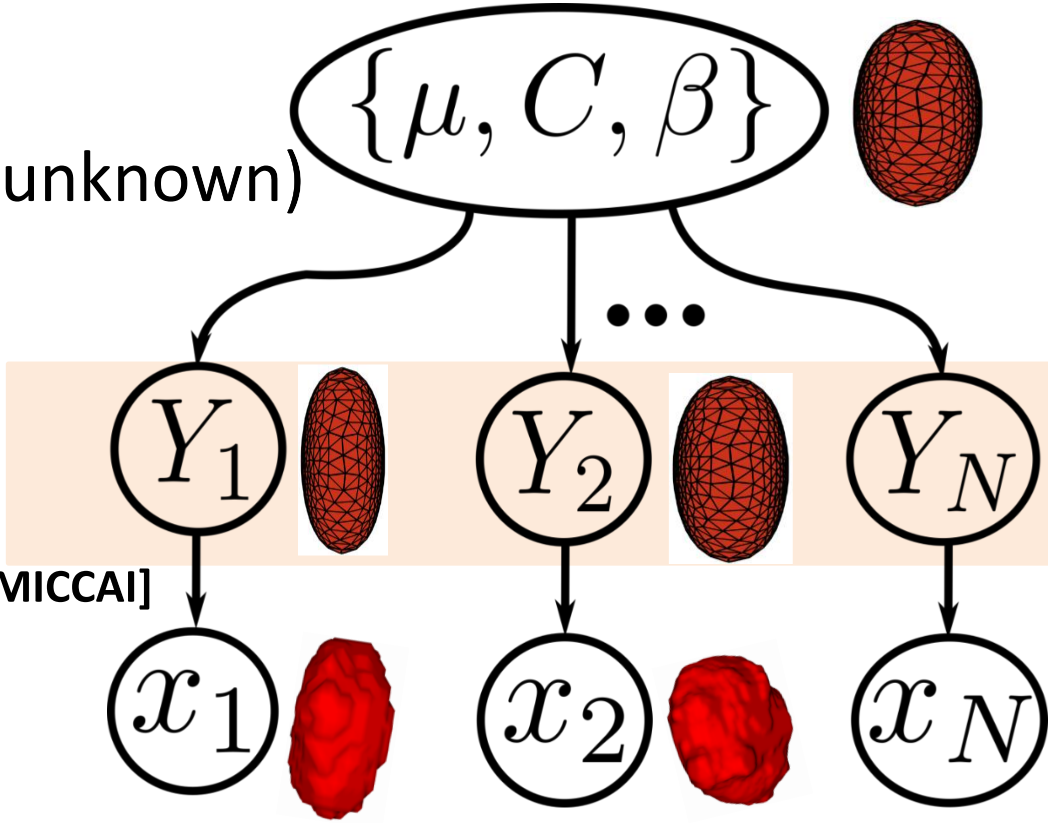

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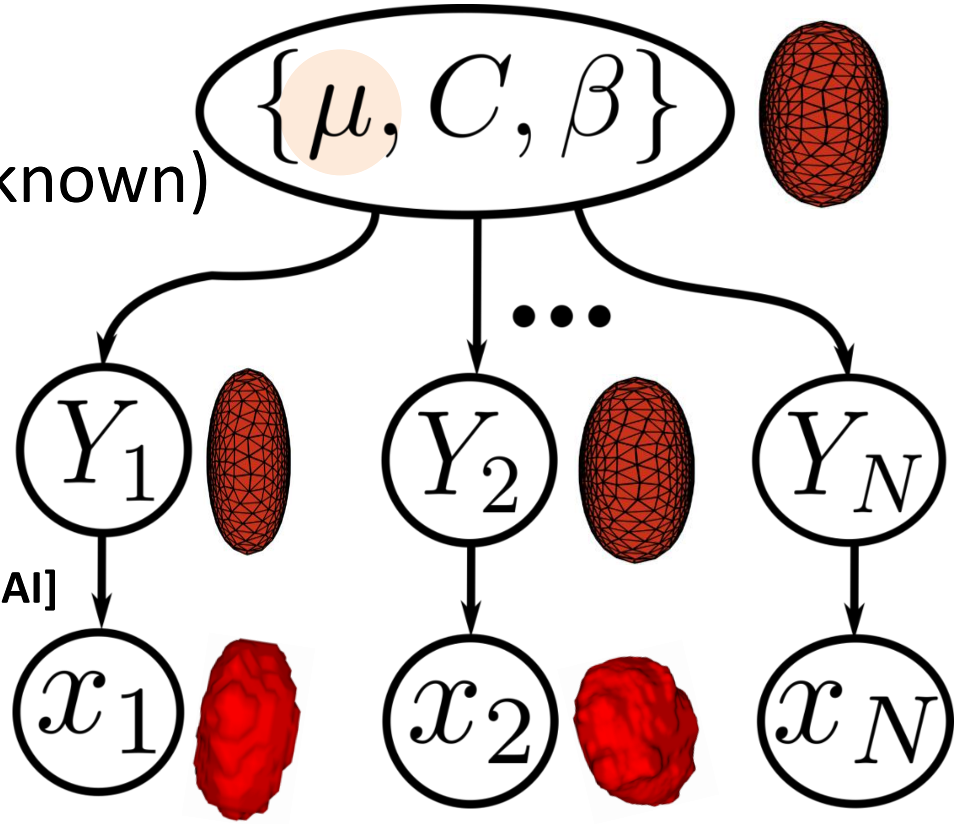

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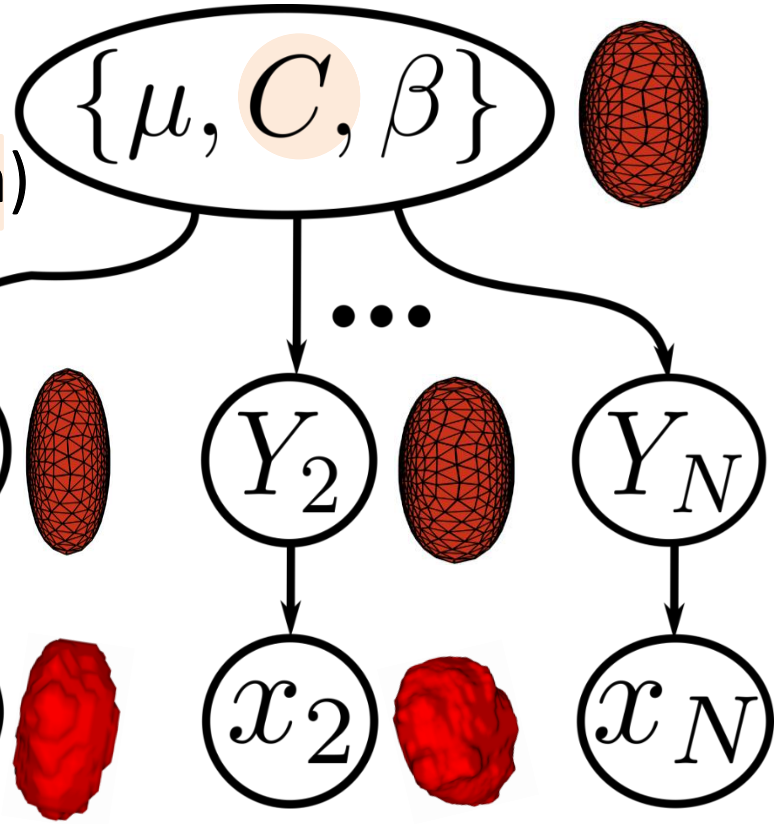

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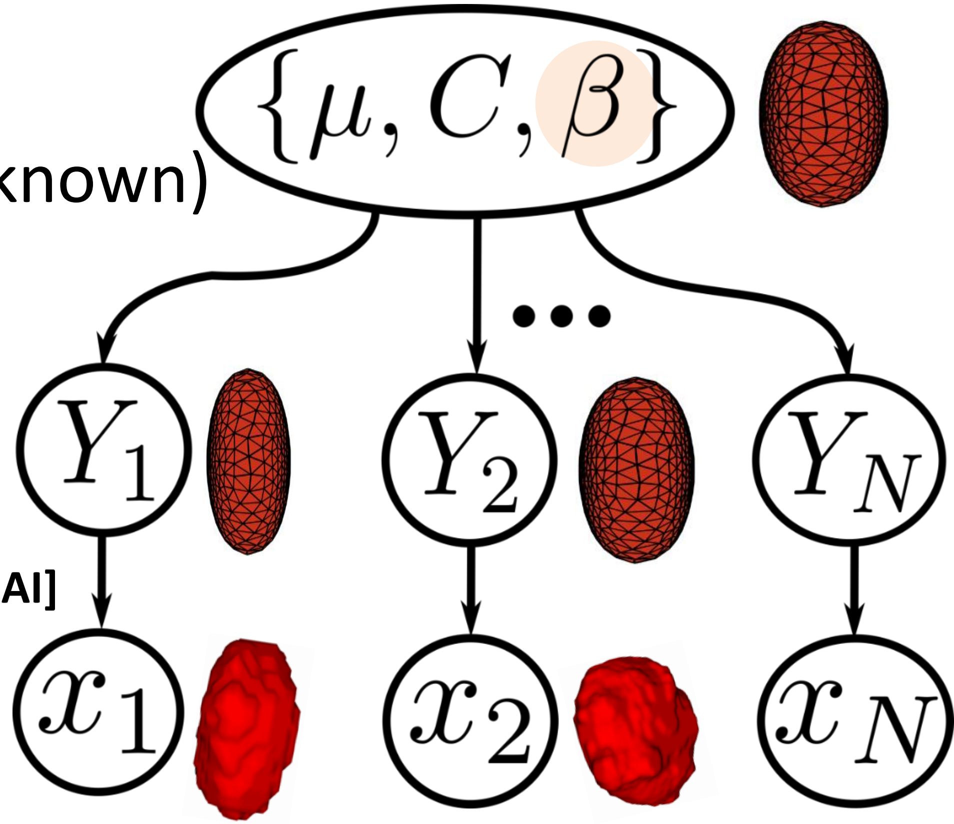
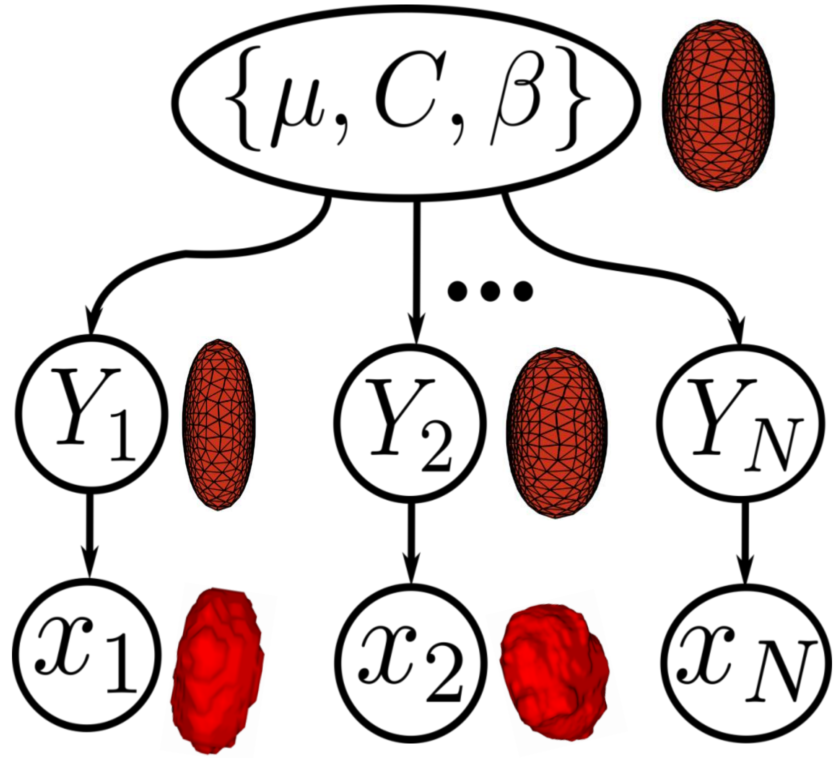


[A Gaikwad, SJ Shigwan, SP Awate, 2015 MICCAI]

Statistical Model on Shapes

- Group
  - Data x
  - Individual shape y
  - Mean μ
  - Modes of variation C
  - Smoothness prior β (free parameter)



$$\theta := \{\mu, C\}$$

$$\max_\theta P(x|\theta) := \max_\theta \int P(x, Y|\theta) dY$$

$$:= \max_{\mu,C} \int P(x|Y) P(Y|\mu, C, \beta) dY$$

Statistical Model on Shapes

- Shape distribution in Riemannian Space
  - Mean μ
  - Covariance C in tangent space at mean

$$P(y_i|\mu, C, \beta) :=$$

$$\frac{1}{\eta(C,\beta)} \exp\left(-\frac{d^2_{\text{Mah}}(y_i;\mu,C)}{2} - \frac{\beta}{2}\sum_{j=1}^{J}\sum_{k\in\mathcal{N}_j}\|y_{ij} - y_{ik}\|_2^2\right)$$

where Mahalanobis distance

$$d^2_{\text{Mah}}(y_i;\mu,C) := \text{Log}_\mu^{\mathbb{S}}(y_i)^\top C^{-1}\text{Log}_\mu^{\mathbb{S}}(y_i)$$

$$\max_{\mu,C}\int P(x|Y)P(Y|\mu,C,\beta)dY$$

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$$\max_{\mu, C} \int P(x|Y) P(Y|\mu, C, \beta) dY$$

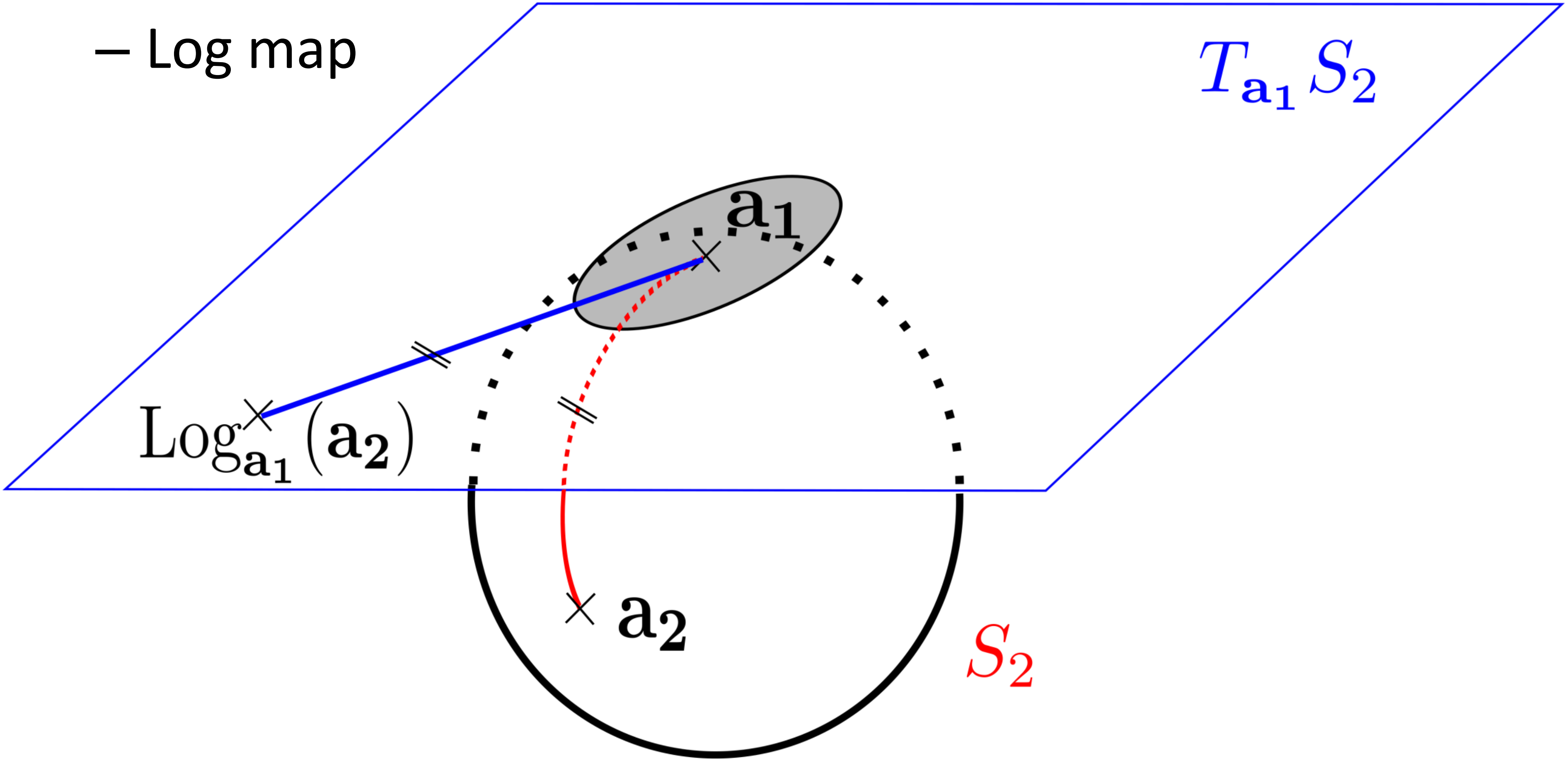
Statistical Model on Shapes

- Shape distribution in Riemannian Space
  - Mean μ
  - Covariance in tangent space at mean C
  - Log map in shape space
    - For pointsets **a**$_1$,**a**$_2$ on unit hypersphere, $\mathrm{Log}_{\mathbf{a}_1}(\mathbf{a}_2)$ is the log map of **a**$_2$ with respect to **a**$_1$
    - $\mathrm{Log}^{\mathbb{S}}_{\mathbf{a}_1}(\mathbf{a}_2) := \mathrm{Log}_{\mathbf{a}_1}(\mathcal{R}^*\mathbf{a}_2)$

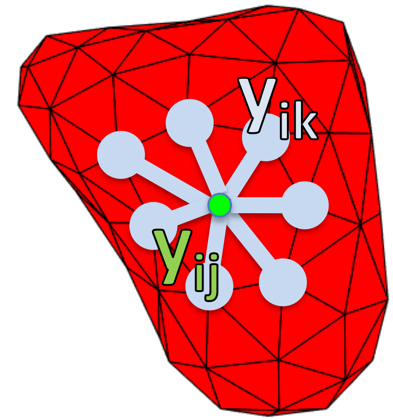
      where, $\mathcal{R}^* := \arg\min_{\mathcal{R}} d_g(\mathcal{R}\mathbf{a}_2, \mathbf{a}_1)$ with
      $\mathcal{R}$ applying rotation to each point within pointset **a**$_2$;
      $d_g(.,.)$ is geodesic distance over unit hypersphere

Statistical Model on Shapes

- Approximate Normal law on hyperspheres
  - Mean
  - Covariance in tangent space at mean
  - Log map



Statistical Model on Shapes



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$$\frac{1}{\eta(C,\beta)} \exp\left(-\frac{d^2_{\mathrm{Mah}}(y_i;\mu,C)}{2} - \frac{\beta}{2}\sum_{j=1}^{J}\sum_{k\in\mathcal{N}_j}\|y_{ij} - y_{ik}\|_2^2\right)$$

  - Smoothness prior on shapes
    - Neighborhood system $\mathcal{N} := \{\mathcal{N}_j\}_{j=1}^{N}$,where $\mathcal{N}_j$ gives set of neighbors of j^{th} point in all $y := \{y_i\}_{i=1}^{N}$

$$\max_{\mu,C}\int P(x|Y)P(Y|\mu,C,\beta)dY$$

Statistical Model on Shapes

- Joint model on individual shapes and individual data
  - Mean
  - Covariance in tangent space at mean
  - Prior Model $\frac{1}{\eta(C,\beta)} \exp\left(-\frac{d^2_{\text{Mah}}(y_i;\mu,C)}{2} - \frac{\beta}{2}\sum_{j=1}^{J}\sum_{k\in\mathcal{N}_j}\|y_{ij}-y_{ik}\|_2^2\right)$
  - Likelihood $P(x_i|y_i) := \exp(-\Delta(x_i,y_i))/\tau$
    - Dissimilarity measure

$$\Delta(x_i,y_i) :=$$
$$\min_{\mathcal{S}_i}\left(\sum_{j=1}^{J}(\mathcal{D}_{x_i}(\mathcal{S}_i y_{ij}))^2 + \sum_{l=1}^{L}\min_j\|\mathcal{Z}^l_{x_i} - \mathcal{S}_i y_{ij}\|_2^2\right)$$

$$\max_{\mu,C}\int P(x|Y)P(Y|\mu,C,\beta)dY$$

Statistical Model on Shapes

- Dissimilarity measure

Where $\mathcal{S}_i$ is similarity transform applied to y_i

$$\Delta(x_i, y_i) :=$$

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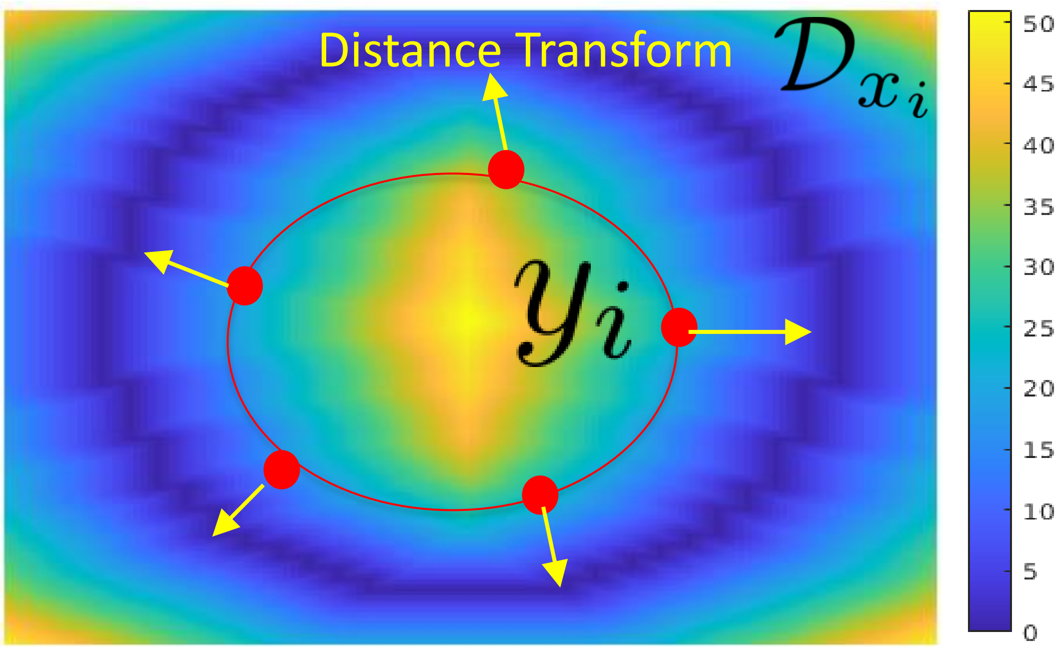

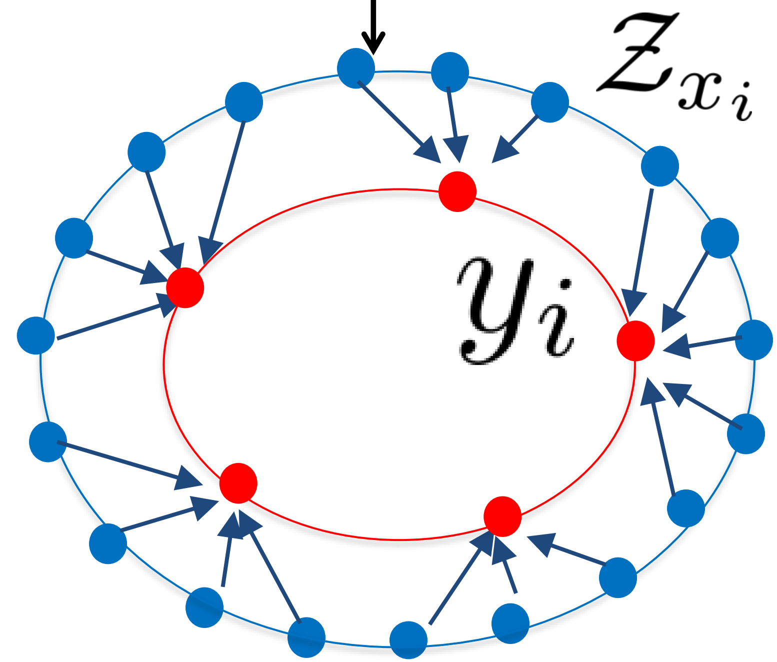

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$$\max_\theta \int P(x, Y|\theta)dY$$

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$$\theta := \{\mu, C\}$$

Statistical Model on Shapes

- We solve $\theta^* := \arg\max_\theta \int P(x, Y|\theta) dY$ using Expectation Maximization (EM)
- At t^{th} iteration $\theta^t := \{\mu^t, C^t\}$
- E step: $\mathcal{Q}(\theta;\theta^t) := E_{P(Y|x,\theta^t)}[\log P(x, Y|\theta)]$
- Expectation is analytically intractable
- We use Monte-Carlo approximation

$$\widehat{\mathcal{Q}}(\theta;\theta^t) \approx \frac{1}{S}\sum_{s=1}^{S} \log P(x, y^s|\theta),$$

$$\text{where } (y^s) \sim P(Y|x, \theta^t)$$

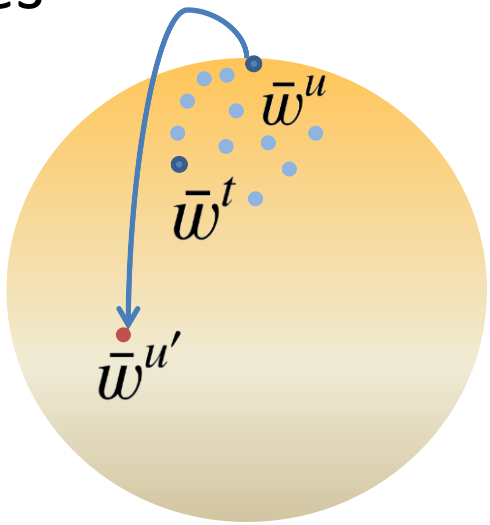
- M step:

$$\theta^{t+1} := \arg\max_\theta \widehat{\mathcal{Q}}(\theta;\theta^t)$$

Statistical Model on Shapes

$$(y^s) \sim P(Y|x, \theta^t)$$

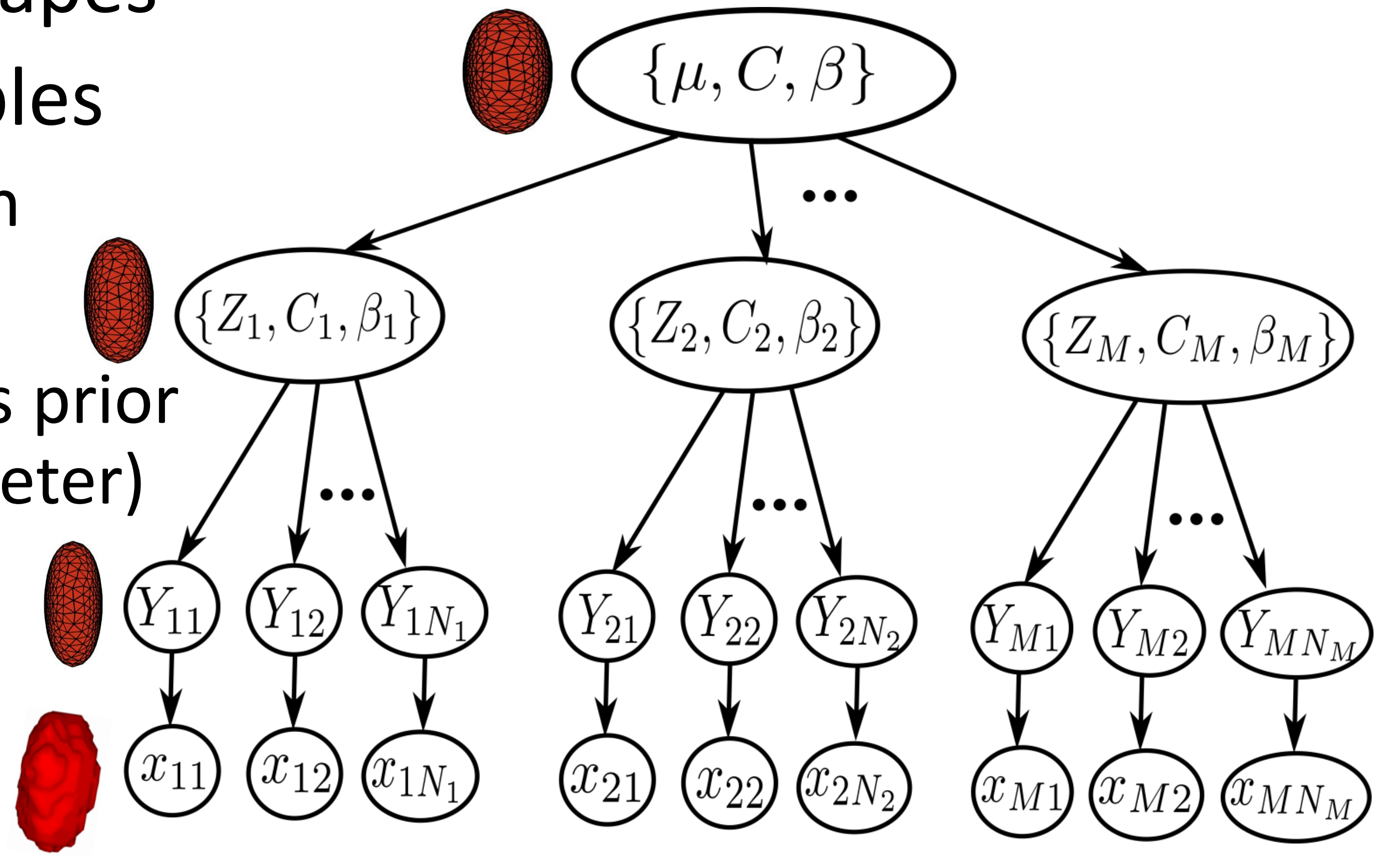
- Leapfrog sampling
  - It maintains a set of state vectors $\bar{w}^u$
  - This set represents independent samples in same distribution $P(\bar{w})$
  - Proposal state is accepted according to Metropolis rule with probability ratio $P(\bar{w}^{u'})/P(\bar{w}^u)$



$$\bar{w}^{u'} := \mathrm{Exp}^{\mathbb{F}}_{\bar{w}^t}(-\mathrm{Log}^{\mathbb{F}}_{\bar{w}^t}(\bar{w}^u))$$

Modeling Multigroup Shape Data

- Data
- Individual shapes
- Group variables
  - Group mean
  - Covariance
  - Smoothness prior (free parameter)
- Population variables
- Used for hypothesis testing [**SJ Shigwan, SP Awate, 2016 MICCAI**]



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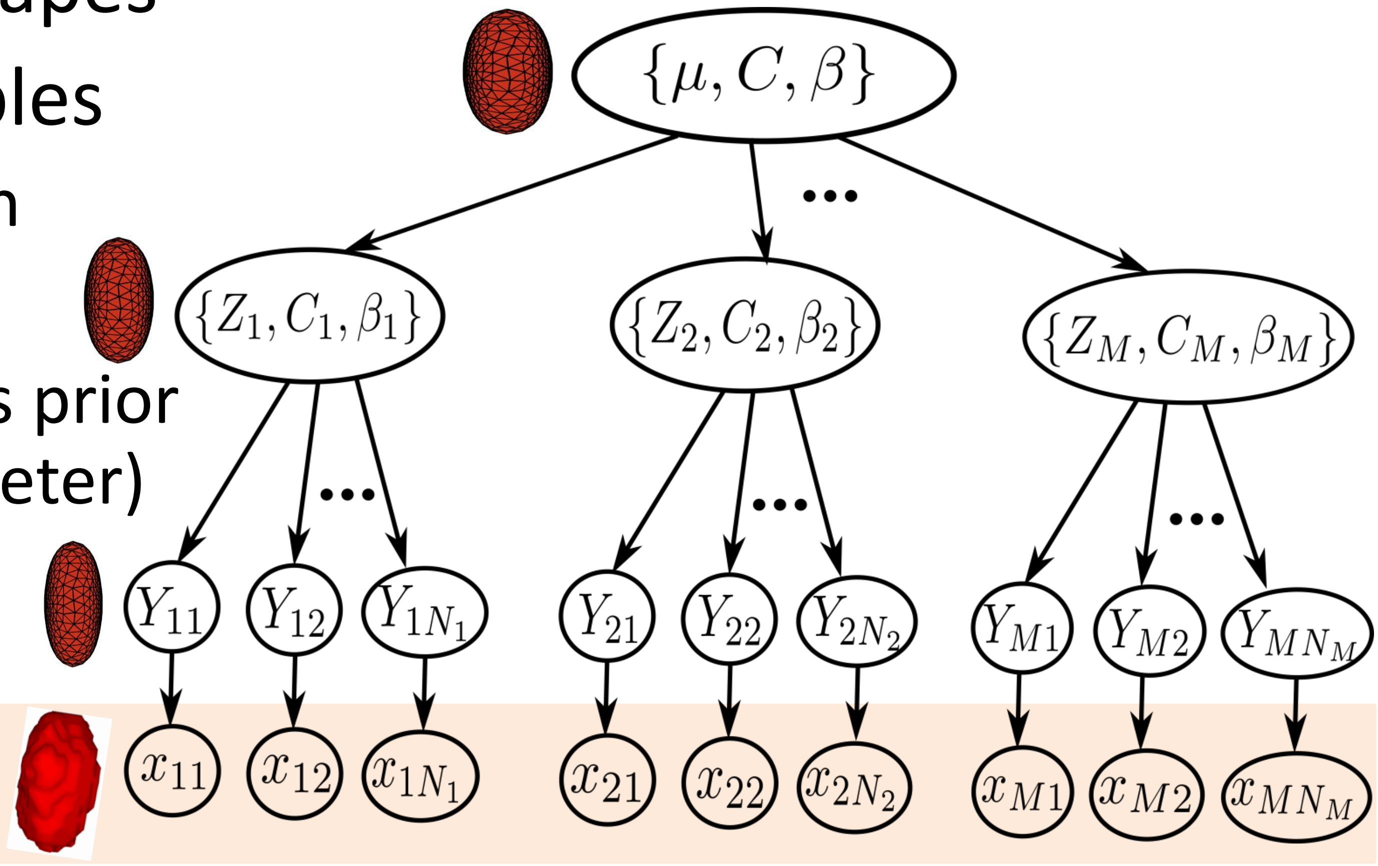

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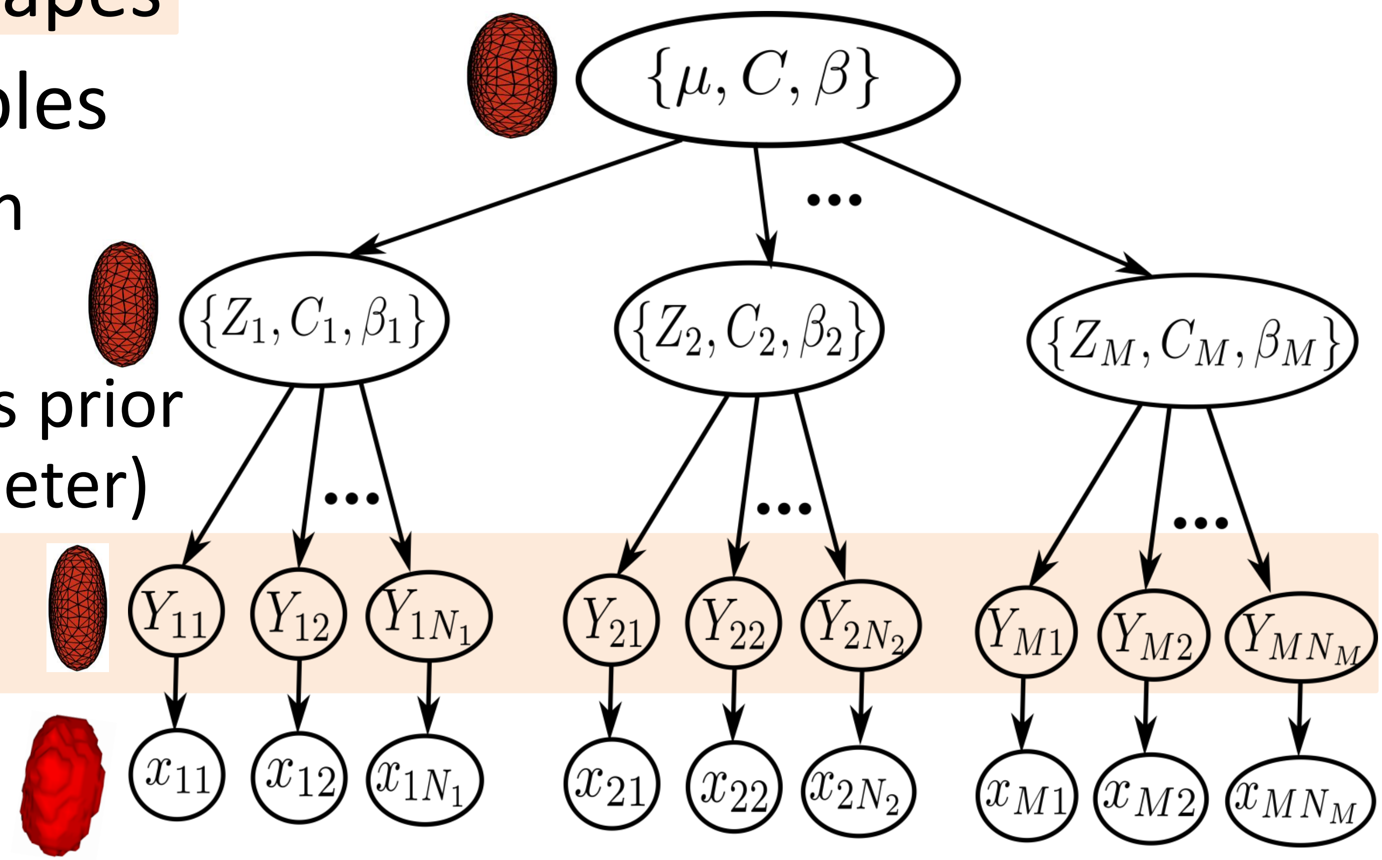

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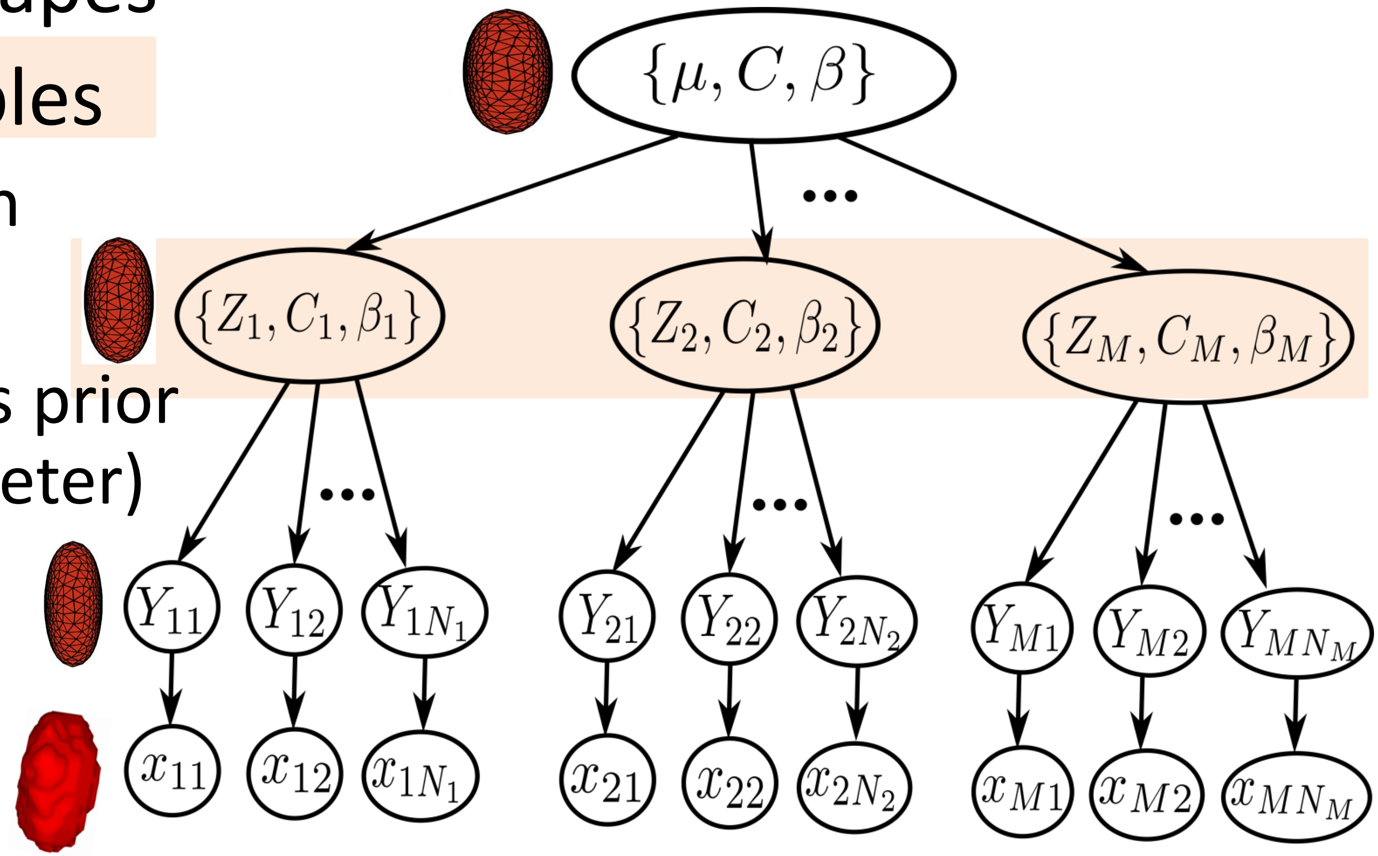

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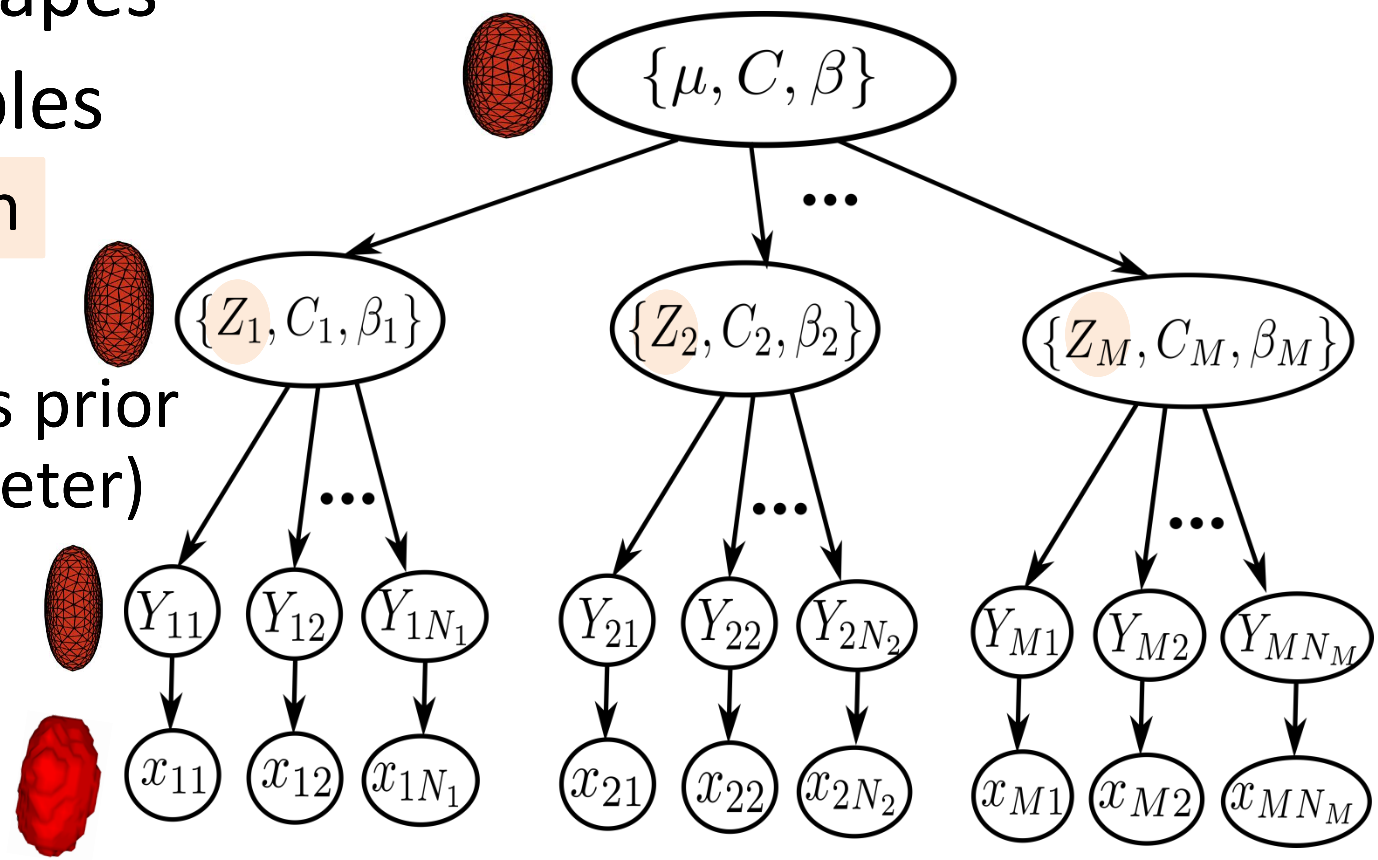

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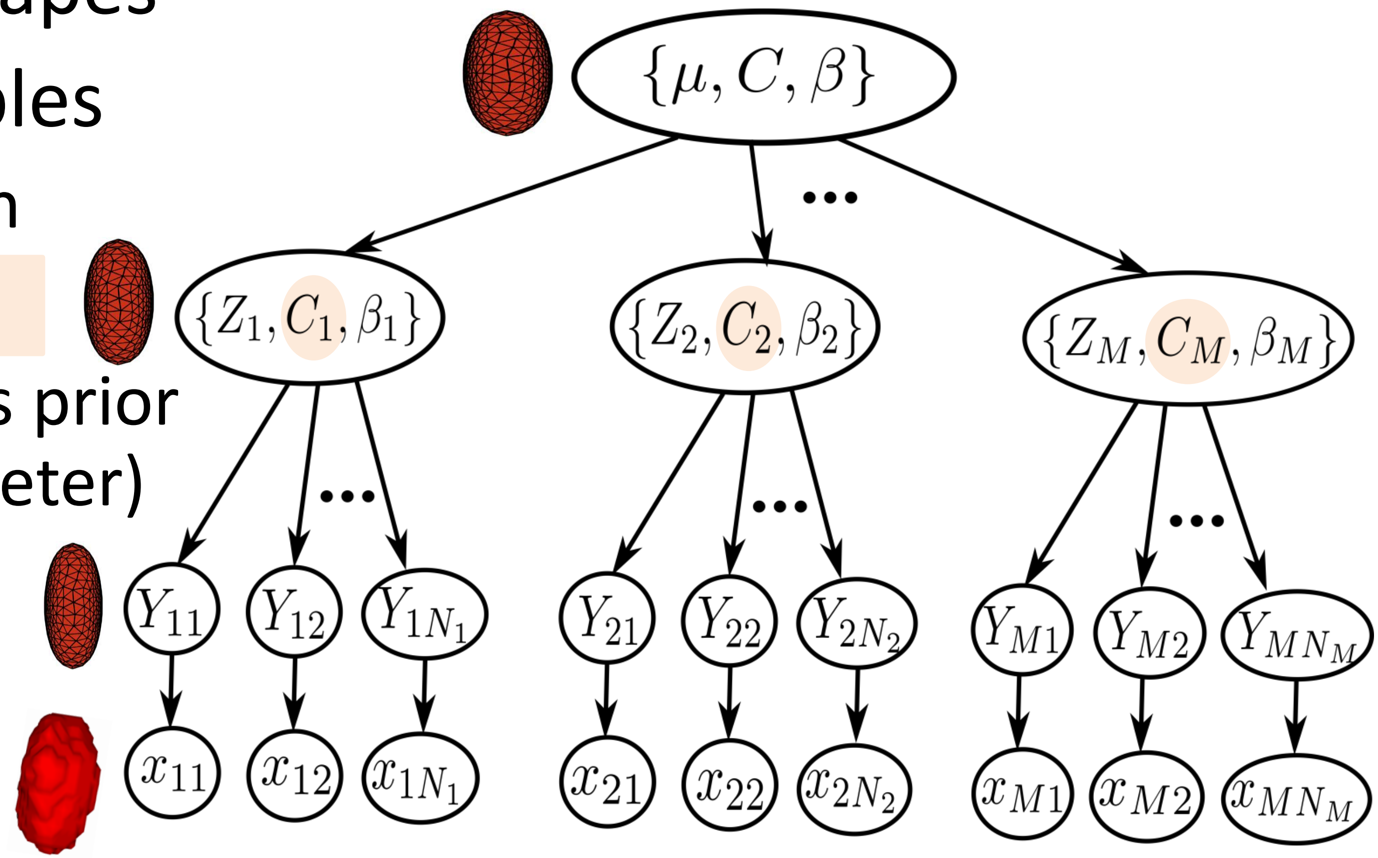

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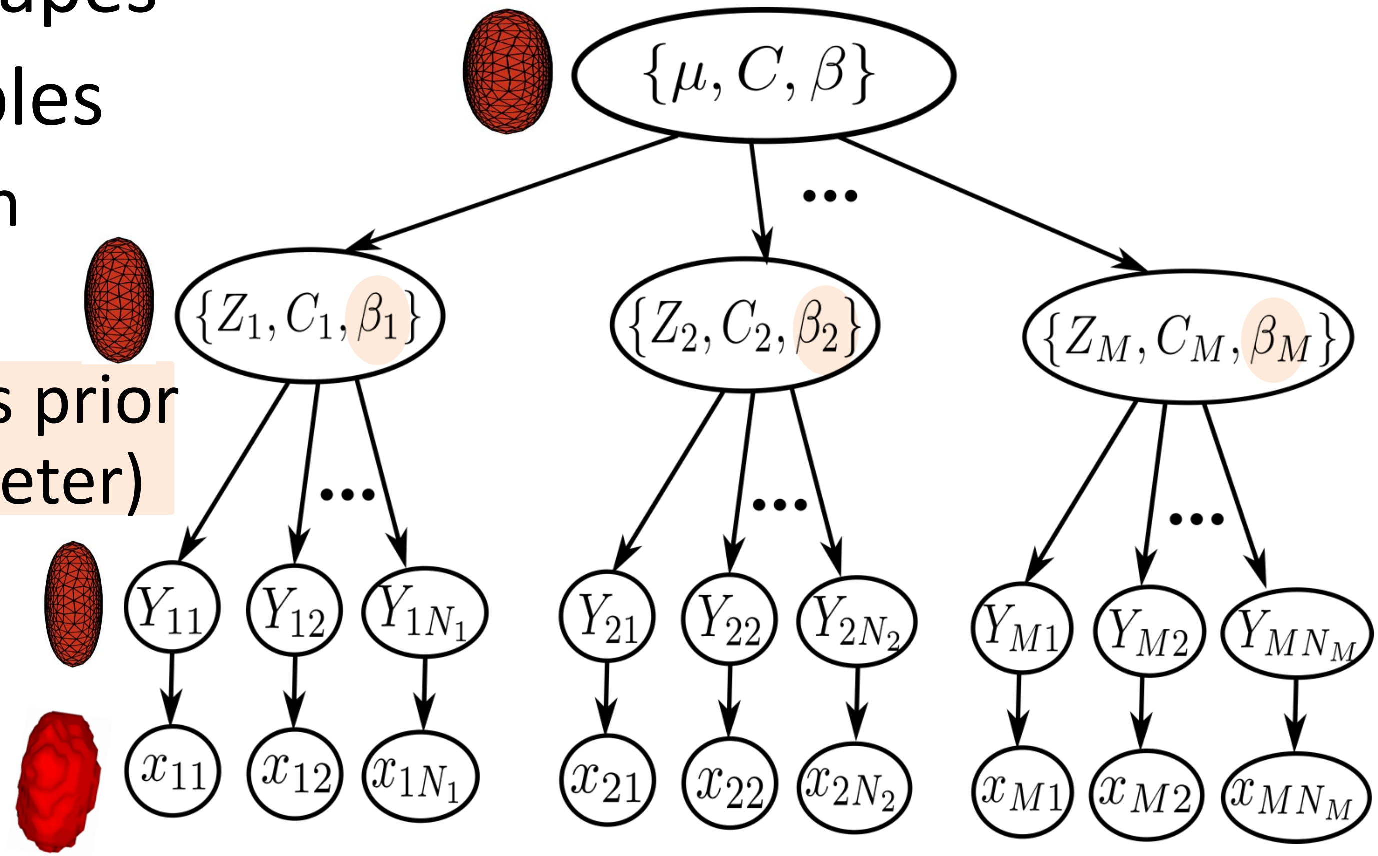

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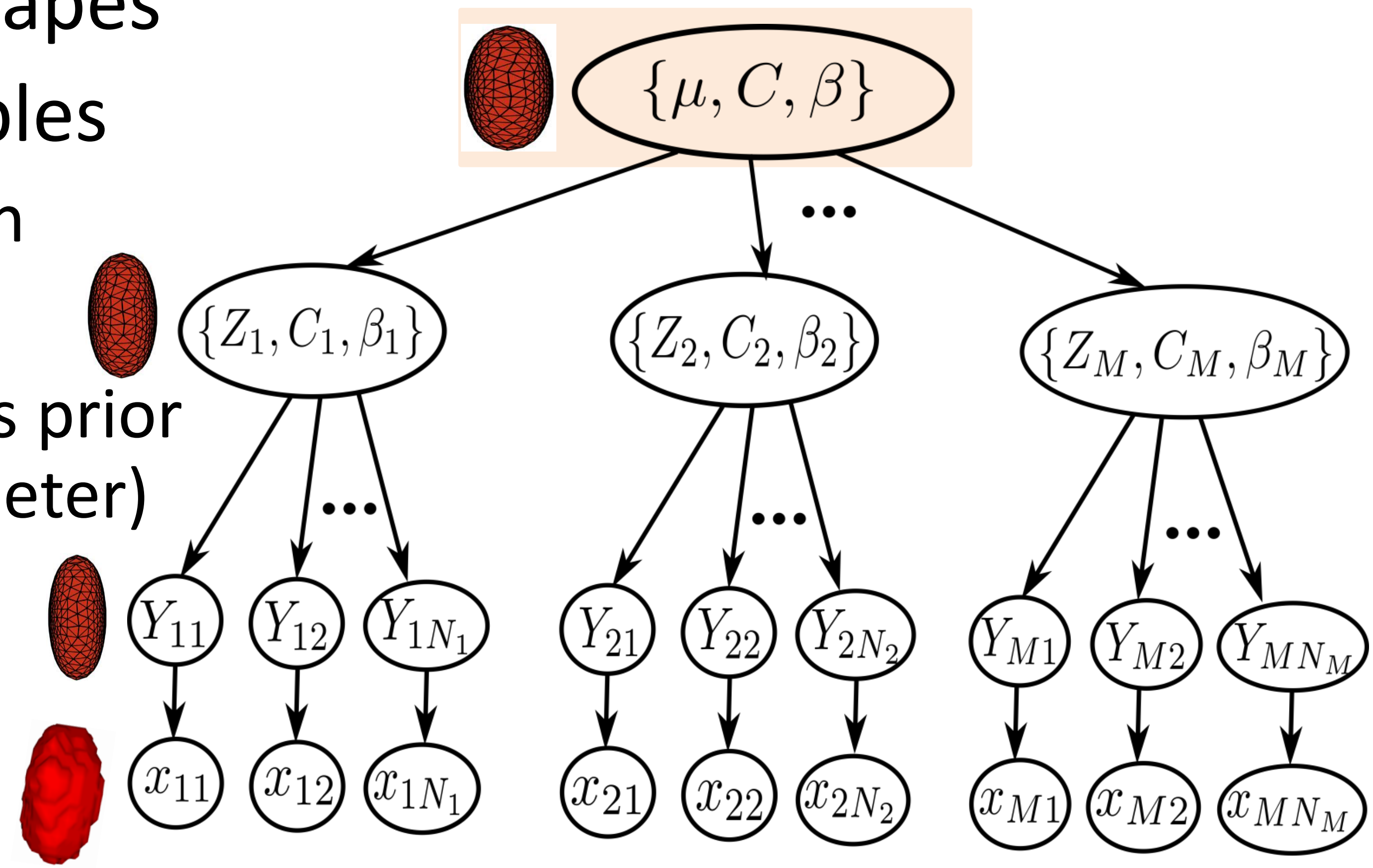
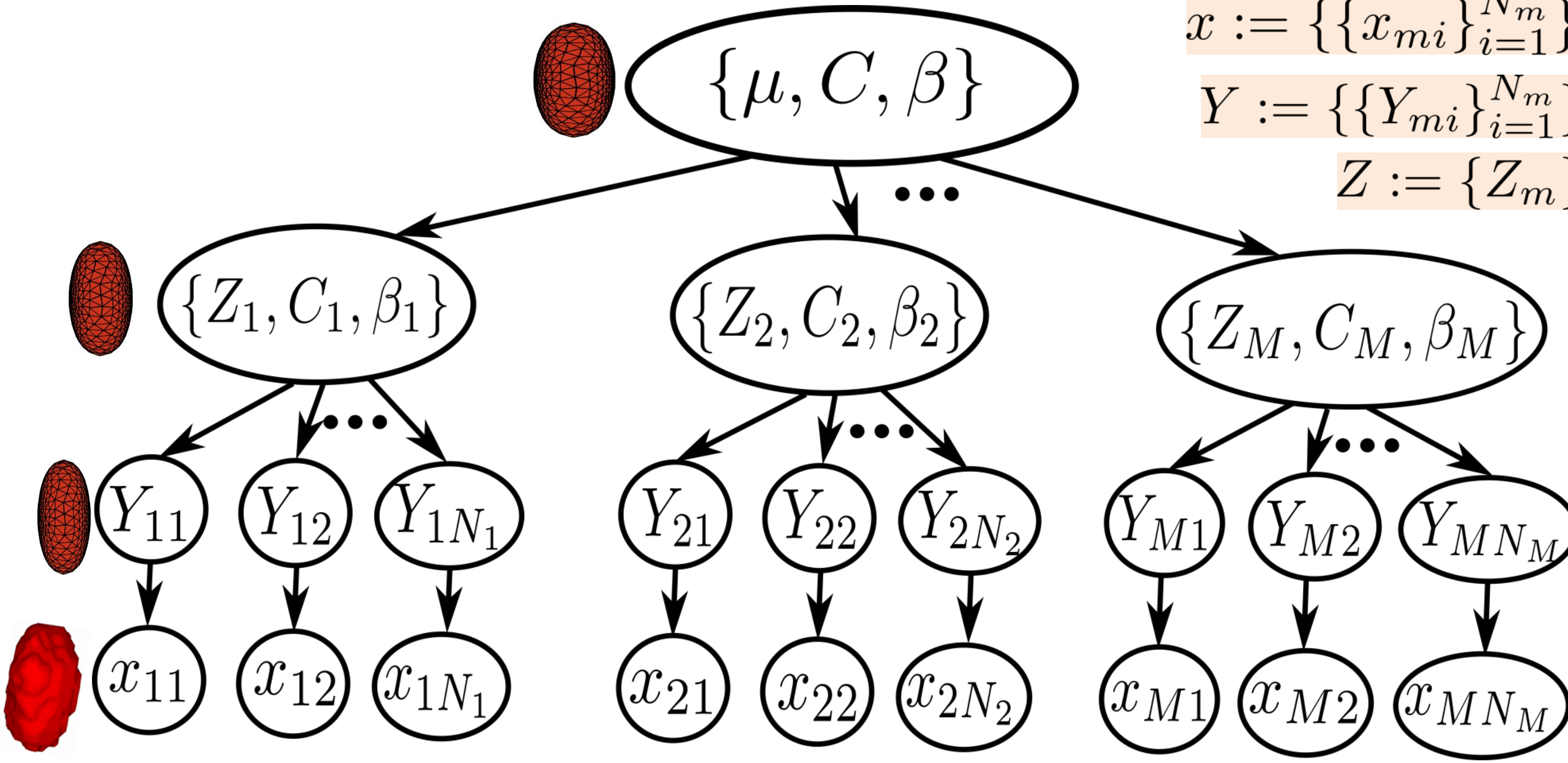


Hierarchical Generative Model



$x := \{\{x_{mi}\}_{i=1}^{N_m}\}_{m=1}^{M}$

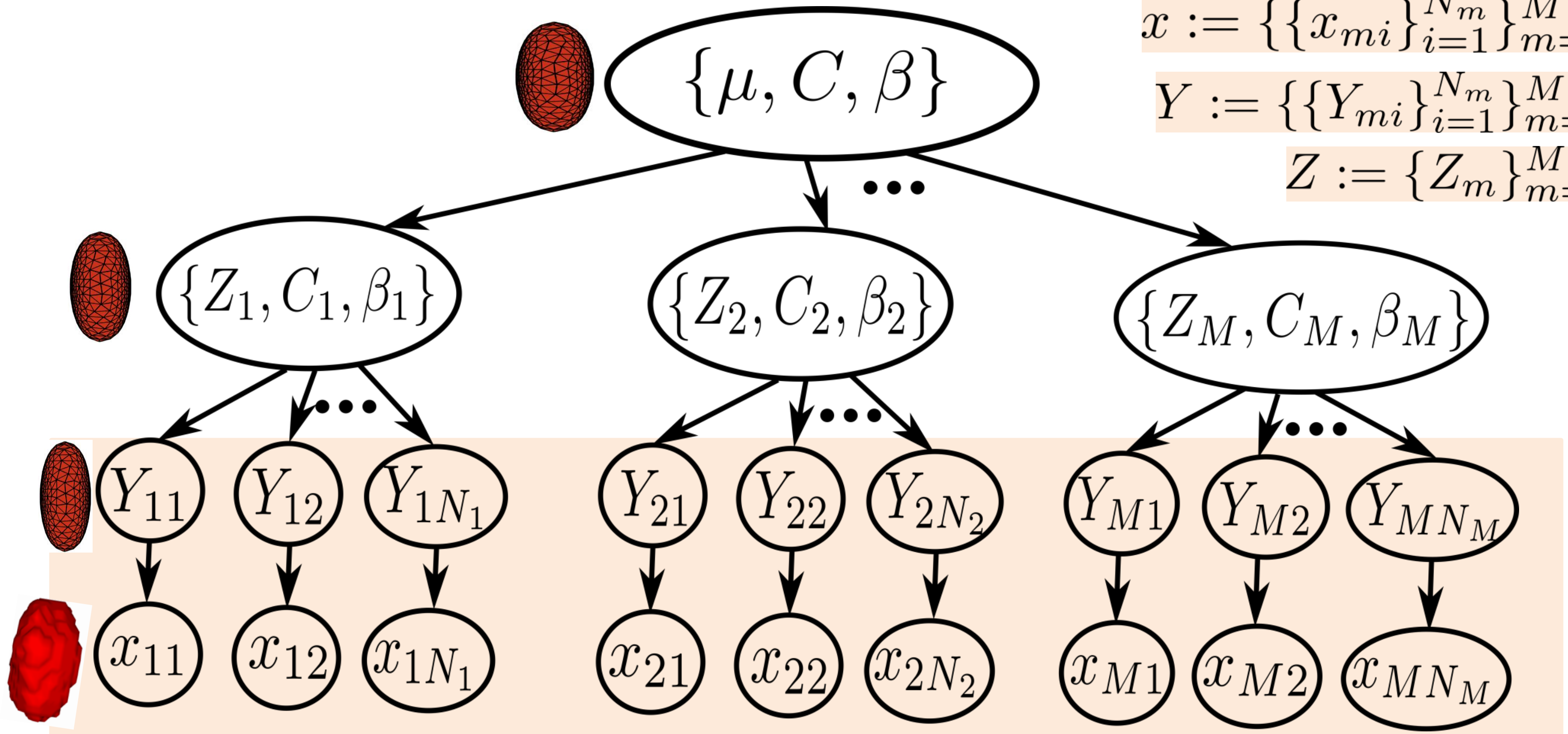
$Y := \{\{Y_{mi}\}_{i=1}^{N_m}\}_{m=1}^{M}$

$Z := \{Z_m\}_{m=1}^{M}$

- With $\theta := \{\mu, C, \{C_m\}_{m=1}^{M}\}$ as parameter and Y and Z as random variables

$$P(x, Y, Z|\theta) := \prod_{m=1}^{M} \prod_{i=1}^{N_M} P(x_{mi}|Y_{mi}) P(Y_{mi}|Z_m, C_m, \beta_m) P(Z_m|\mu, C, \beta)$$

Hierarchical Generative Model



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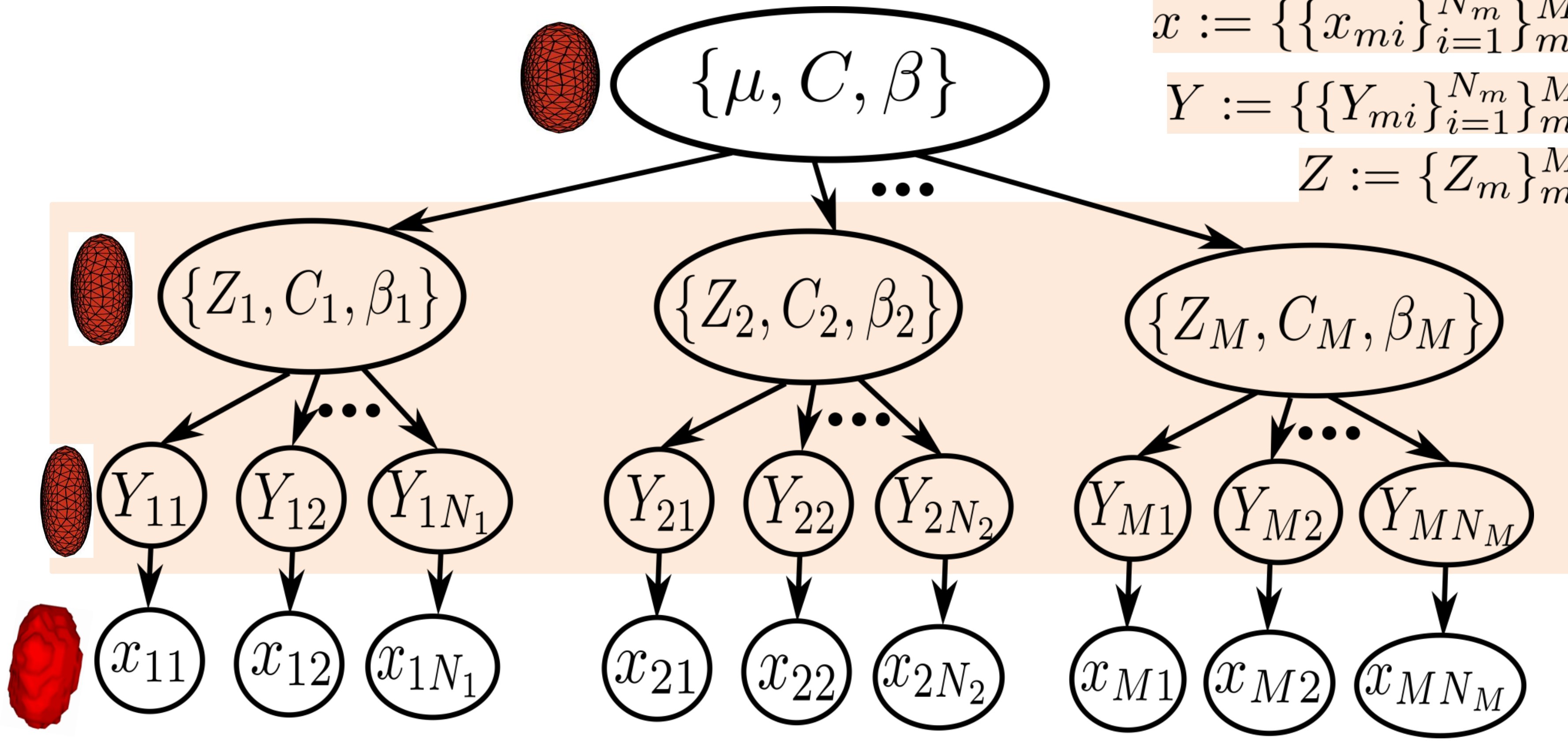

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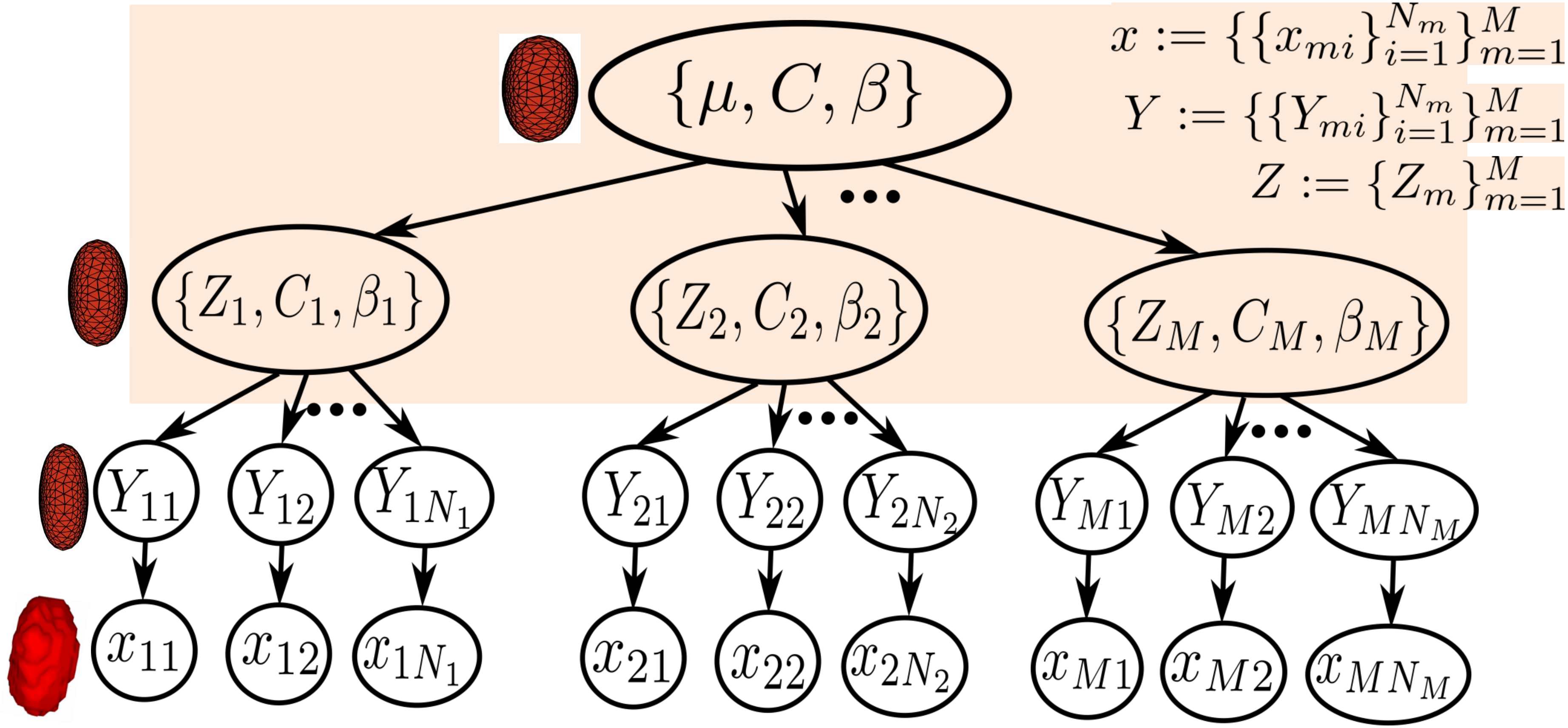

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Hierarchical Generative Model

- We solve $\theta^* := \arg\max_\theta \iint P(x, Y, Z|\theta) dY\, dZ$ using Expectation Maximization (EM)
- At t^{th} iteration $\theta^t := \{\mu^t, C^t, \{C_m^t\}_{m=1}^M\}$
- E step: $Q(\theta; \theta^t) := E_{P(Y,Z|x,\theta^t)}[\log P(x, Y, Z|\theta)]$
- Expectation is analytically intractable
- We use Monte-Carlo approximation
  $\widehat{Q}(\theta; \theta^t) \approx \frac{1}{S} \sum_{s=1}^S \log P(x, y^s, z^s|\theta)$,
  where $(y^s, z^s) \sim P(Y, Z|x, \theta^t)$
- M step:

$$\theta^{t+1} := \arg\max_\theta \widehat{Q}(\theta; \theta^t)$$

Hypothesis Testing

- Null hypothesis: Given 2 groups of data *A* and *B* are from the same distribution
- We do **permutation testing** to test null hypothesis of equality of two group distribution in shape space
  - Permutation test is non-parametric, robust to type-1 errors
- Proposed test statistic T to measure differences between two shape distributions

$$T := \frac{1}{N_A} \sum_{i=1}^{N_A} d^2_{\mathrm{Mah}}(y_{Ai}; z^B, C^B) + \frac{1}{N_B} \sum_{i=1}^{N_B} d^2_{\mathrm{Mah}}(y_{Bi}; z^A, C^A)$$

Hypothesis Testing

- Null hypothesis: Given 2 groups of data *A* and *B* are from the same distribution
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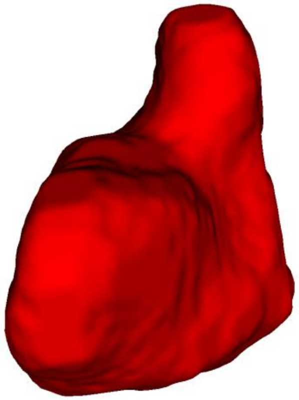
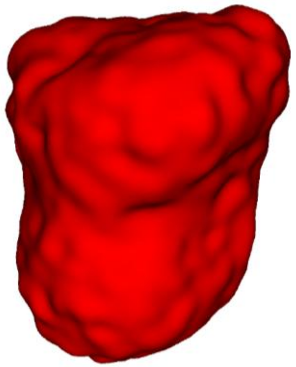
Results on Anatomical Data

- Data from 2 groups of bones in humans
  - Group 1(Males) and Group 2(Females)
    - Each group has 15 individuals
    - Manually segmented images, imperfect segmentations

Courtesy of Kevin E. Burroughs, MD



Hamate



Capitate

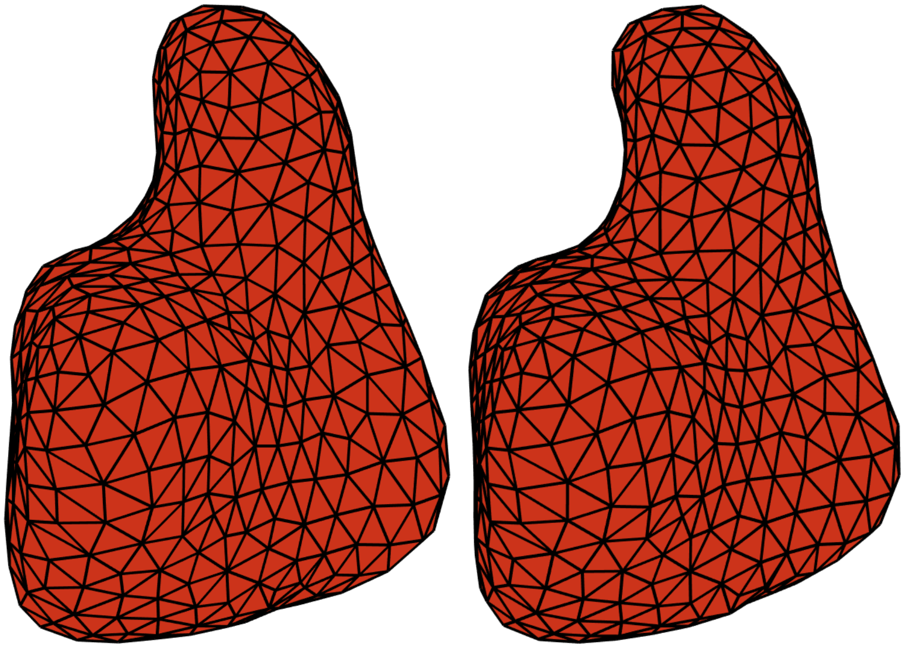
Evaluation - Baseline

- ShapeWorks [Cates 2017]
  - 3D pointset-based framework for shape modeling
  - Does not employ hierarchical model
  - Forces point locations (within shape) to object boundary
  - Does not enforce shape smoothness and shape alignment during model fitting

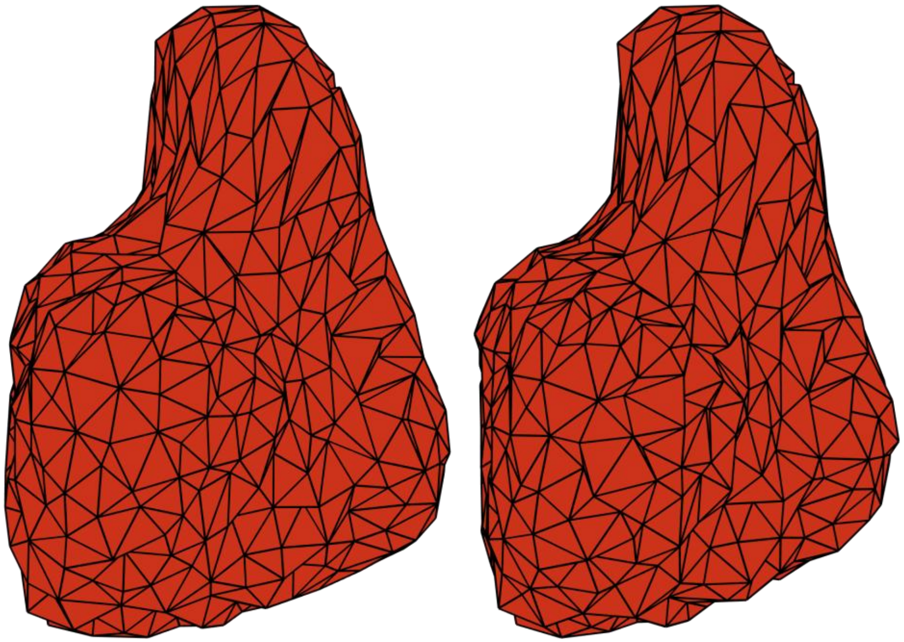
[Cates 2017] Joshua Cates, Shireen Elhabian, Ross Whitaker. "Shapeworks: particle-based shape correspondence and visualization software." Statistical Shape and Deformation Analysis. Academic Press, 2017. 257-298

Results on Anatomical Data-Hamate

Group means z_1, z_2



Ours

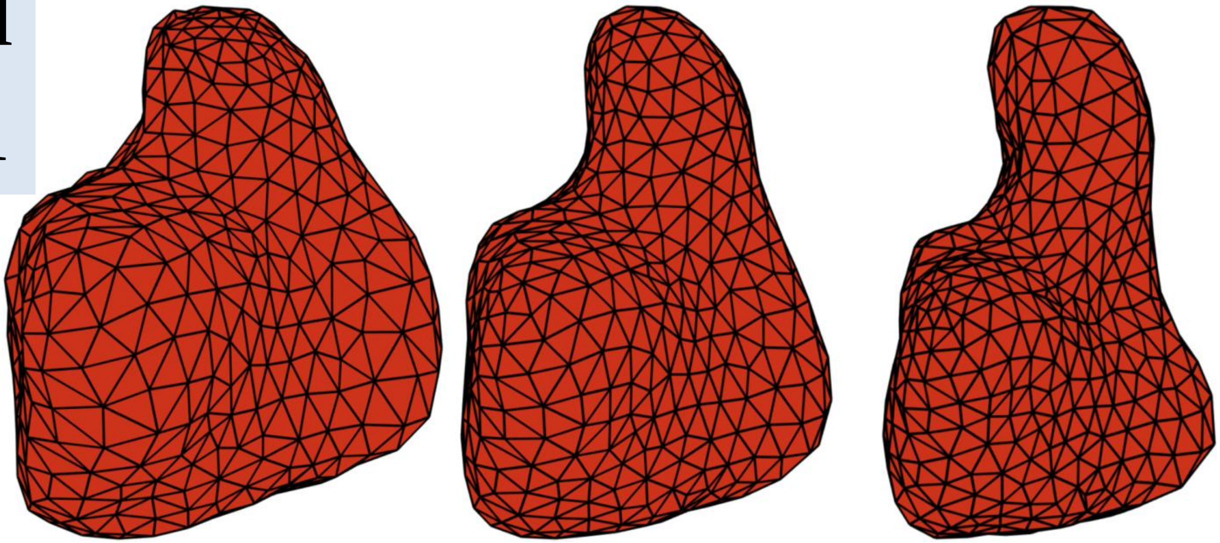


ShapeWorks

Results on Anatomical Data-Hamate

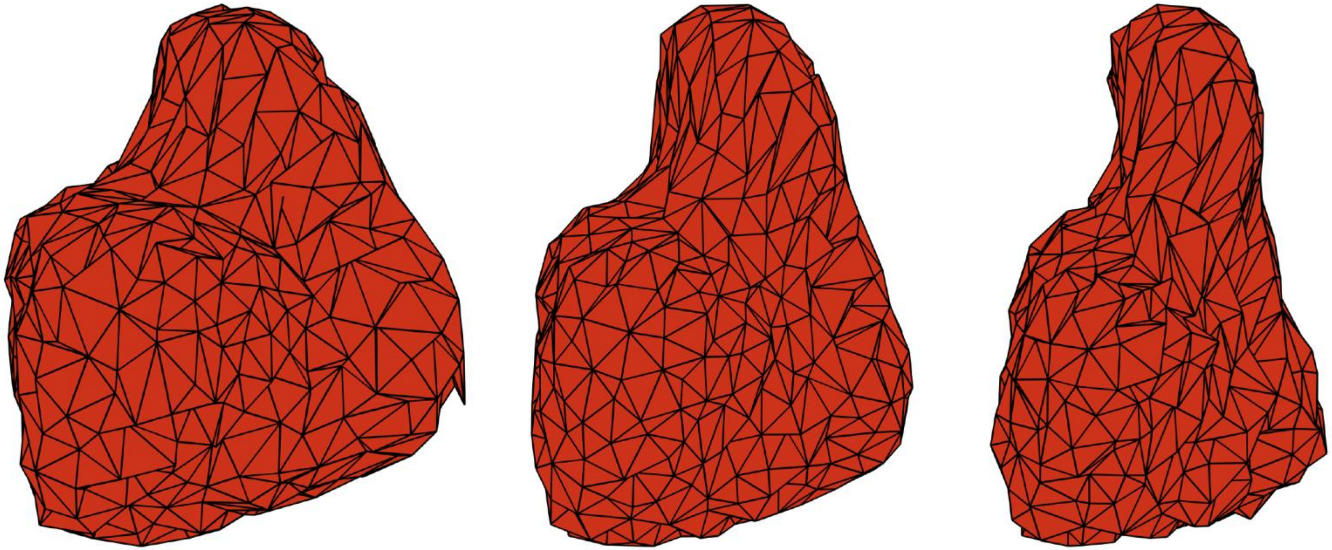
Variation around mean of group-1

Ours



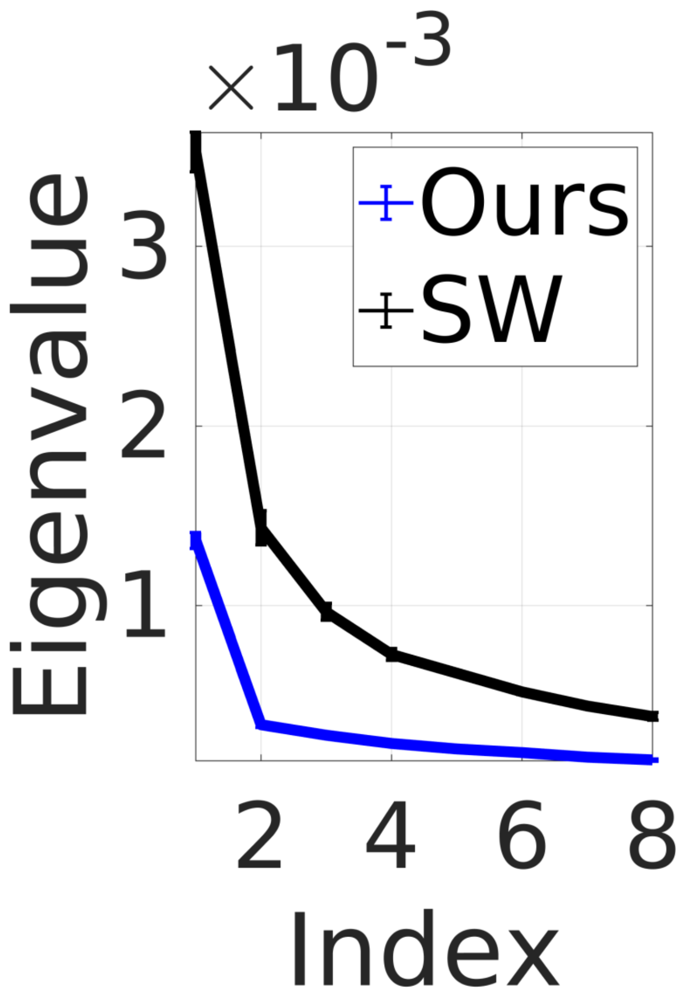
Mean shape z_1

ShapeWorks

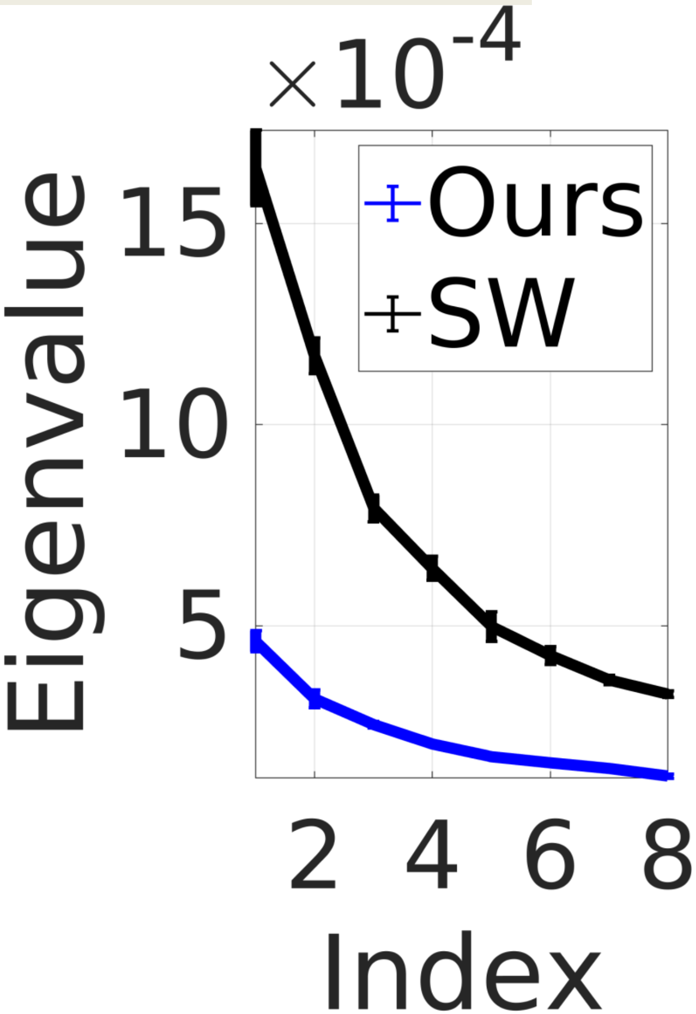


Results on Anatomical Data-Hamate

Eigenvalues spectra comparison



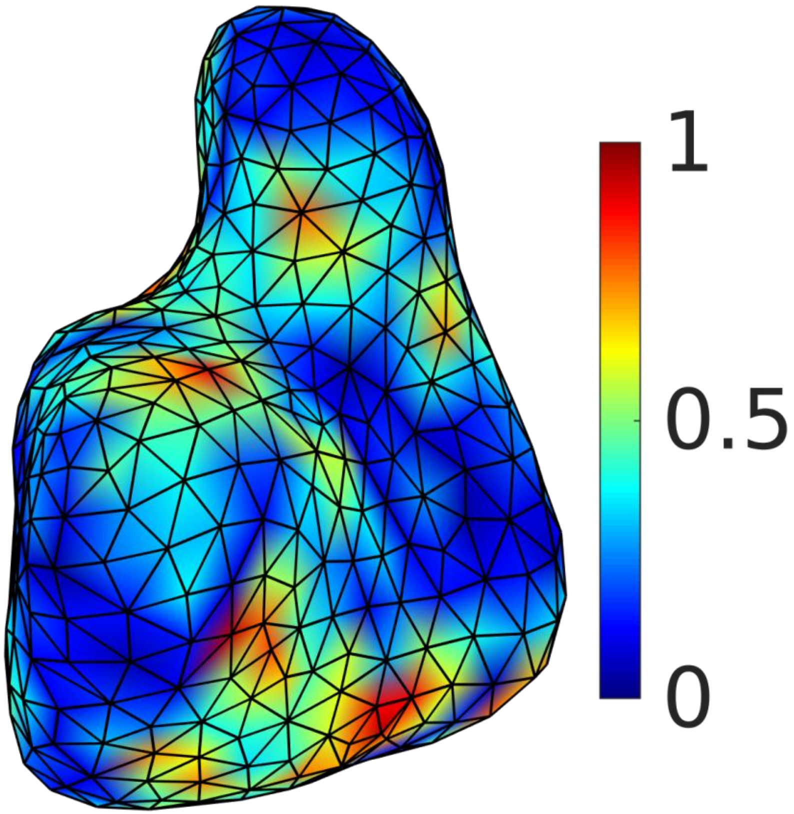
Group Variance C_1



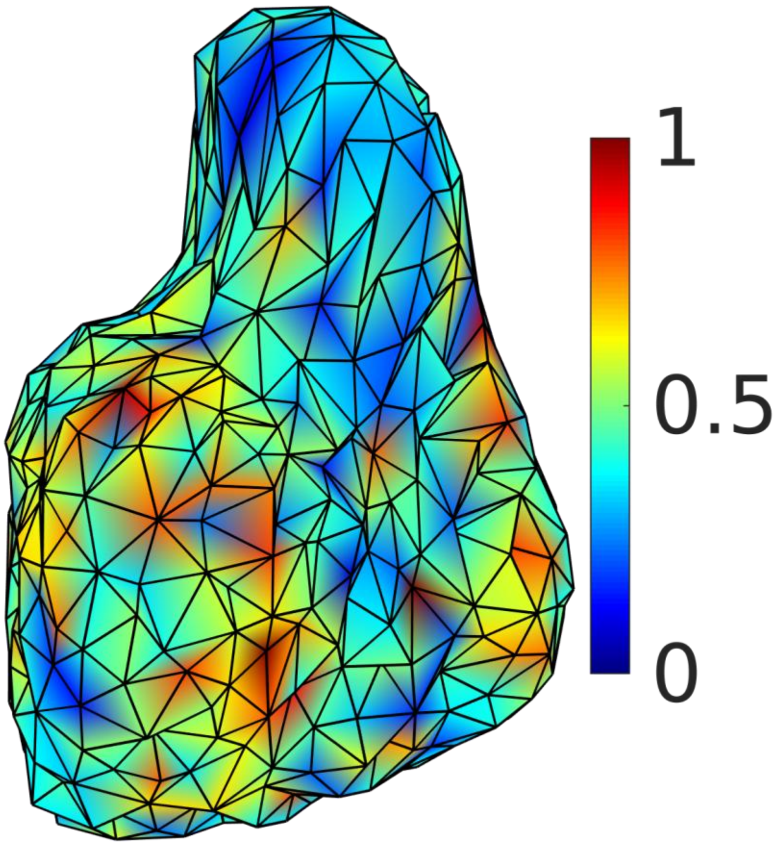
Group Variance C_2

Results on Anatomical Data-Hamate

Population mean μ with Cohen’s d effect size



Ours



ShapeWorks

Results on Anatomical Data-Hamate

- Permutation test
  - Test statistic does not follow standard probability distribution
  - Infer distribution using permutation test
- Hypothesis testing results
  - Our method give p-value 0.13 and Shapeworks give p-value 0.3
  - Our method more confident about rejecting null hypothesis that given two groups are from same distribution, than Shapeworkss

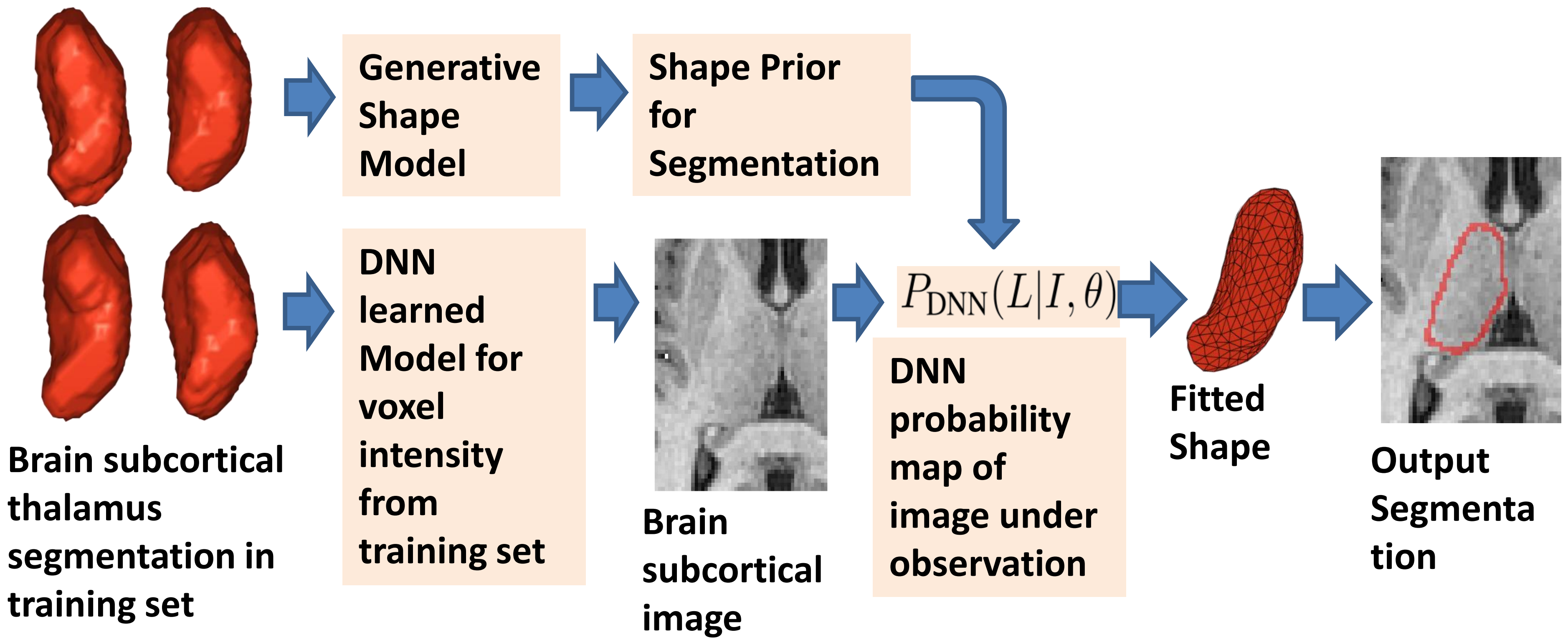
Segmentation using shape prior

Segmentation using shape prior

- Shape model provides shape mean and covariance matrix
- Eigenvectors of covariance matrix could act as basis for shape distribution in shape space
- We have designed a novel objective function for finding optimized segmentation

Segmentation using shape prior

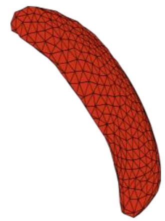
- Data
  - Object segmentations in a training set. Test image.



- Task
  - Learning a generative model of shape
  - Estimating segmentation using shape prior

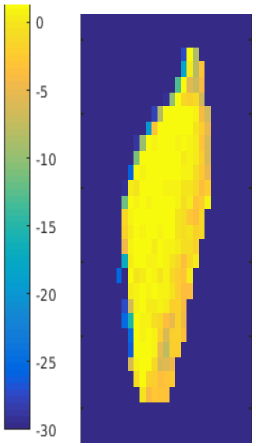
Segmentation using shape prior

- Similarity measure between a feature $\boldsymbol{F}$ of image I and shape surface y
  - Shape surface y



  - $\mathcal{S} \circ y$ = similarity transform of shape surface y

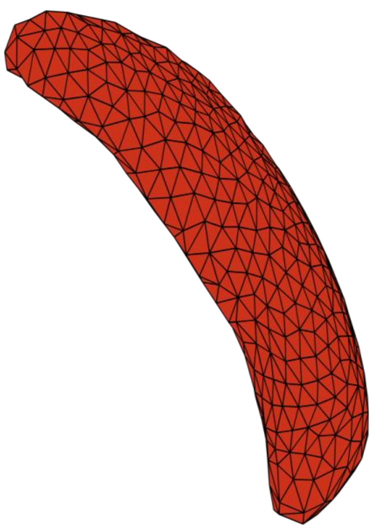
  - $\mathcal{B}(\cdot)$ = binarized volume for shape y
  - $L_x \in \{0, 1\}$ be label at voxel x
    - 1 = object's interior
    - 0 = object's exterior
  - $P_{\mathrm{DNN}}(L|I, \theta)$ = distribution trained using DNN with weight parameters θ



$\log P_{\mathrm{DNN}}(l_x = 1|\theta, I)$

Segmentation using shape prior

- PCA based shape representation with prior
  - $\Lambda := \{\lambda_k\}_{k=1}^{K}$ = Top *K* eigenvalues of covariance matrix C
  - $V := \{v_k\}_{k=1}^{K}$ = Top *K* eigenvectors of covariance matrix C
  - Any point y in shape space, $y := \mathrm{Exp}_\mu(\sum_{k=1}^{K} w_k v_k)$
  - $\{w_k \in \mathbb{R}\}_{k=1}^{K}$ are top *K* basis coefficients
  - τ is weighting parameter to sparsity prior and ζ is normalizing constant



Regularizing prior on coefficients $\{w_k \in \mathbb{R}\}_{k=1}^{K}$

$$P(y = \mathrm{Exp}_\mu(\textstyle\sum_{k=1}^{K} w_k v_k)) := \zeta \exp(-\tau \textstyle\sum_{k=1}^{K} |w_k| / \sqrt{\lambda_k})$$

Segmentation using shape prior

- Log-posterior PDF of object shape y

$$\log P(y = \mathrm{Exp}_{\mu}(\textstyle\sum_{k=1}^{K} w_k v_k) | \mathcal{S}, w, \mu, \Lambda, V, \theta, I)$$

$$:= \textstyle\sum_{x \in \mathcal{X}} \mathcal{B}(\mathcal{S} \circ y)_x \log P_{\mathrm{DNN}}(l_x = 1 | \theta, I) +$$

$$(1 - \mathcal{B}(\mathcal{S} \circ y)_x) \log P_{\mathrm{DNN}}(l_x = 0 | \theta, I) - \tau \textstyle\sum_{k=1}^{K} |w_k| / \sqrt{\lambda_k}$$

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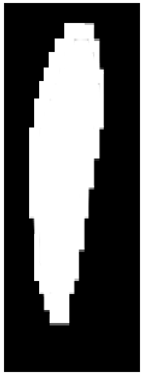

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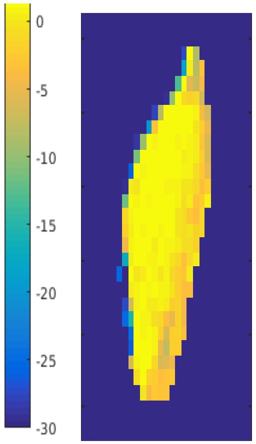

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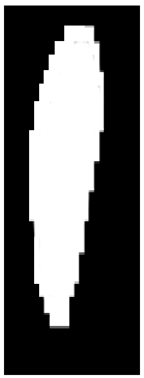

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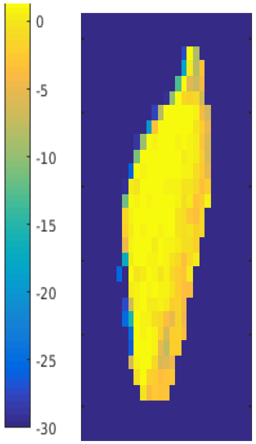

- This sum of multiplications is nothing but sum of log-probabilities of overlapping voxels.

Segmentation using shape prior

- Log-posterior PDF of object shape y

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- This is equivalent term for background voxels.

Segmentation using shape prior

- Log-posterior PDF of object shape y

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$$:= \sum_{x \in \mathcal{X}} \mathcal{B}(\mathcal{S} \circ y)_x \log P_{\mathrm{DNN}}(l_x = 1 | \theta, I) + (1 - \mathcal{B}(\mathcal{S} \circ y)_x) \log P_{\mathrm{DNN}}(l_x = 0 | \theta, I) - \tau \sum_{k=1}^{K} |w_k| / \sqrt{\lambda_k}$$

- This last term is log of regularizing prior on basis coefficients.

Segmentation using shape prior

- Log-posterior PDF of object shape y

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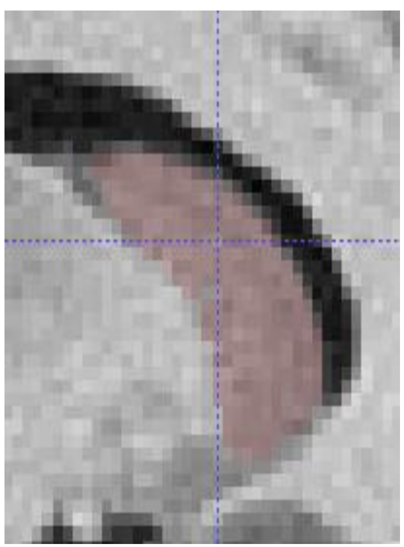
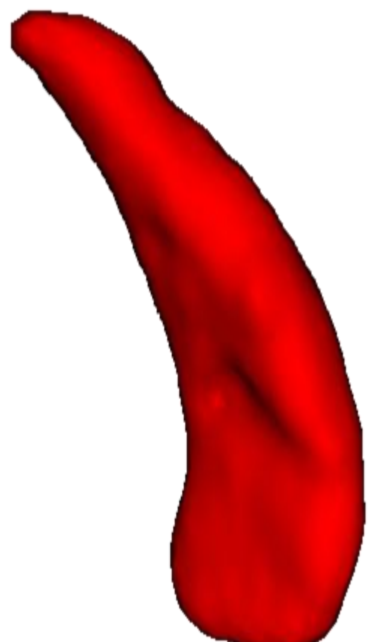
- Optimization of objective function (log-posterior PDF)
  - Calculating gradient is complex
  - Represent all parameters as set of independent scalars
  - Do parameter update by local unidirectional interval search for next update iteratively till convergence

Segmentation Results: Subcortical Brain Structures

- Data from 30 human individuals (small number)
  - 30 human subjects
  - Low-quality expert segmentations
  - Actual MRI images for segmentation is available

Caudate Slice



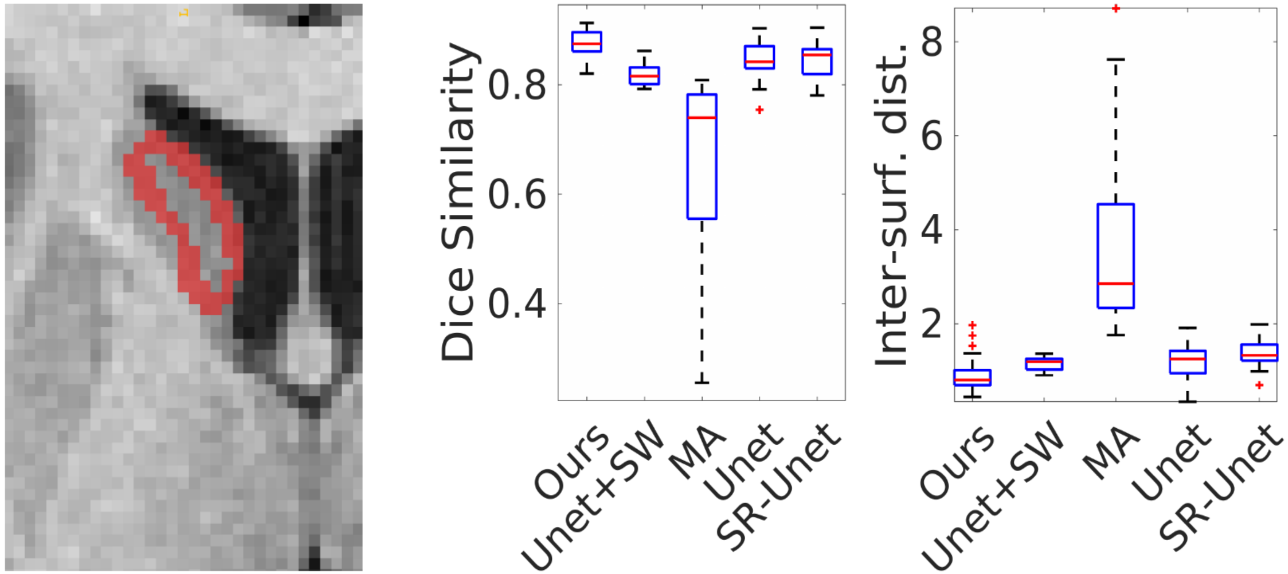


Caudate 3D

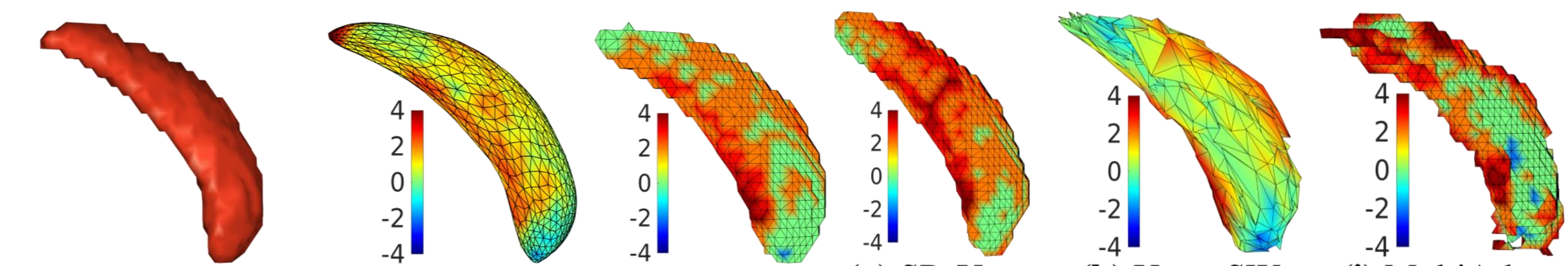
Segmentation Results: Subcortical Brain Structures

- Experiment
  - Data: Brain MRI image with object segmentations
  - Baselines
    - **U-Net**: 3D U-Net [Brox 2015 MICCAI]
    - **SR-Unet**: Shape regularized Unet [Ravishankar 2017 MICCAI]
    - **Unet+SW**: U-Net coupled with shape prior learned from ShapeWorks [Cates 2017 Stat. Shape Deformation Analysis]
    - **MA**: Multiatlas segmentation using nonlinear nonparametric diffeomorphic registration
  - Comparison between estimated and true segmentations
- Evaluation metrics
  - Dice Similarity Coefficient (DSC)
  - Inter-surface distance
    - Histogram of nearest neighbor distances between surface pointsets of both images

Segmentation Results: Caudate

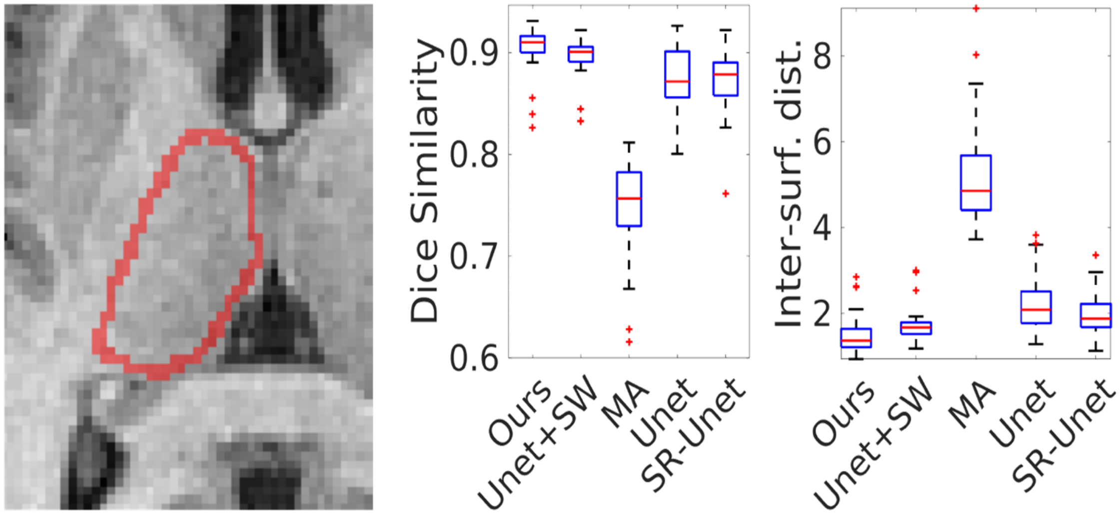


(a) Evaluation Data **(b)** **(c)**



(d) High-Qual. Truth [DSC = 1, Dist. = 0] **(e) Ours** [0.86,0.88] **(f)** Unet [0.82,1.55] **(g)** SR-Unet [0.82,1.7] **(h)** Unet+SW [0.81,1.23] **(i)** MultiAtlas [0.77,2.54]

Segmentation Results: Thalamus



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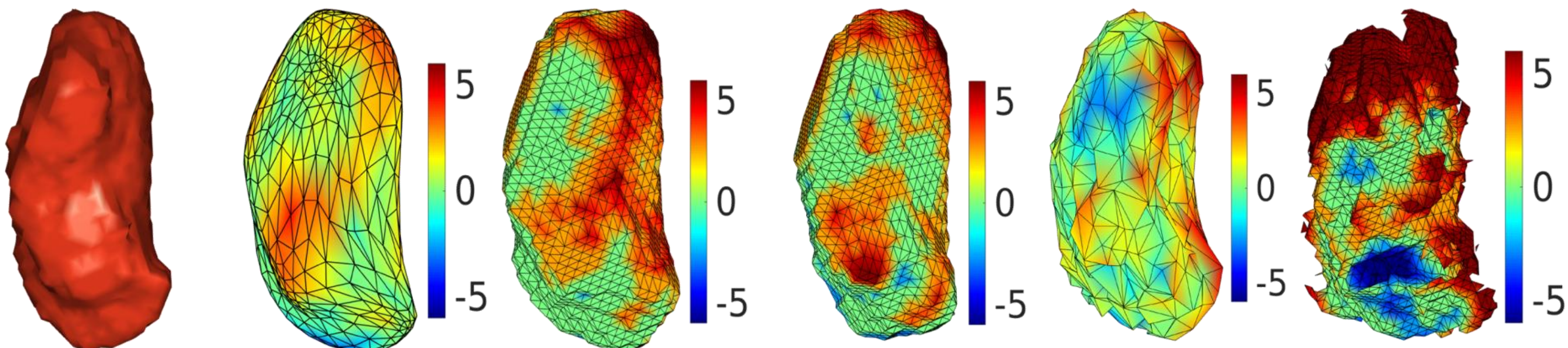
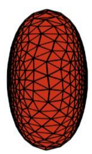
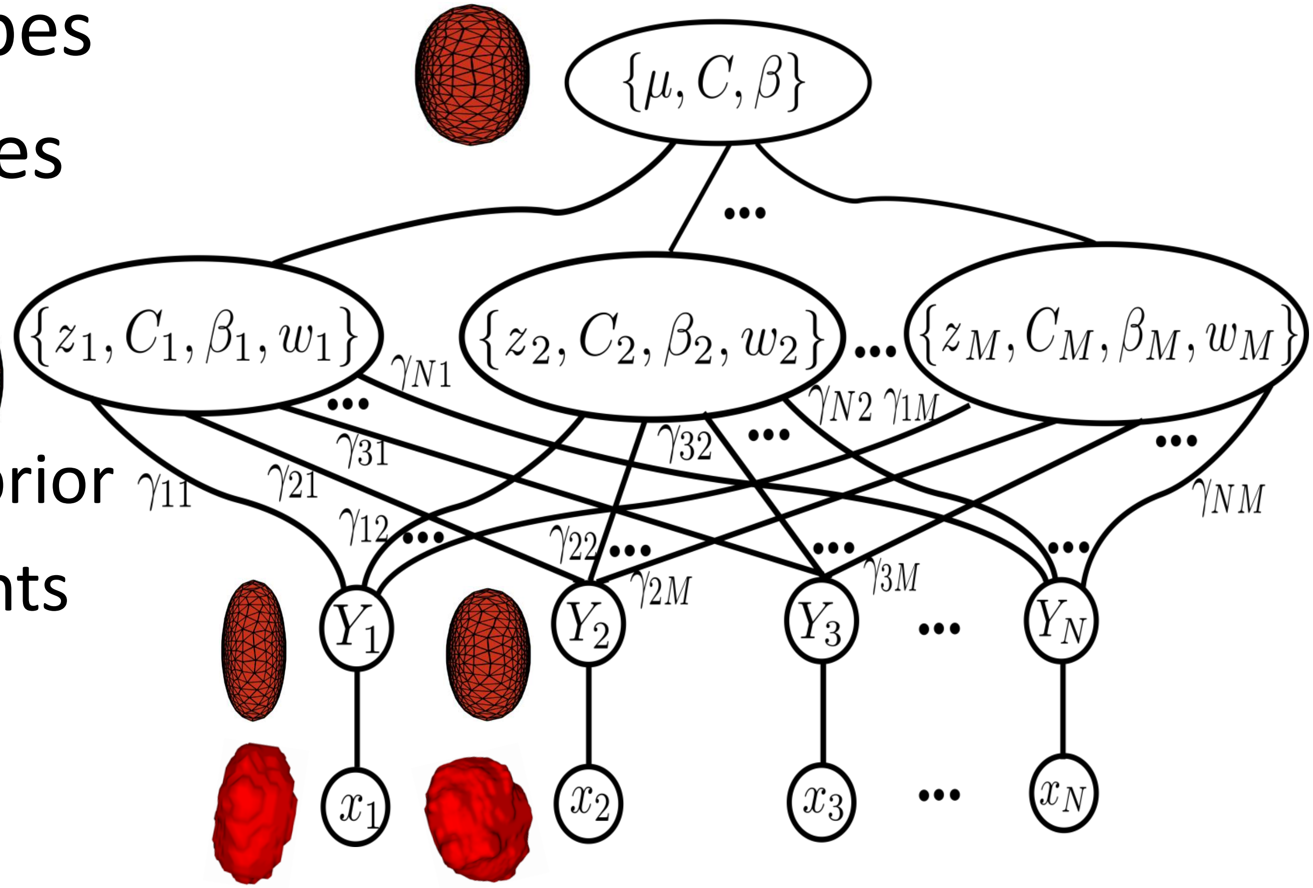


(d) High-Qual. Truth [DSC = 1, Dist. = 0] **(e) Ours** [0.91,1.48] **(f)** Unet [0.88,2.16] **(g)** SR-Unet [0.89,1.97] **(h)** Unet+SW [0.90,1.68] **(i)** MultiAtlas [0.75,5.07]

Clustering using hierarchical Riemannian Shape model

Modeling Clusters of Shapes

- Data
- Individual shapes
- Cluster variables
  - Cluster mean
  - Covariance 
  - Smoothness prior
  - Mixture weights
  - Membership
  - Cluster labels
- Population variables



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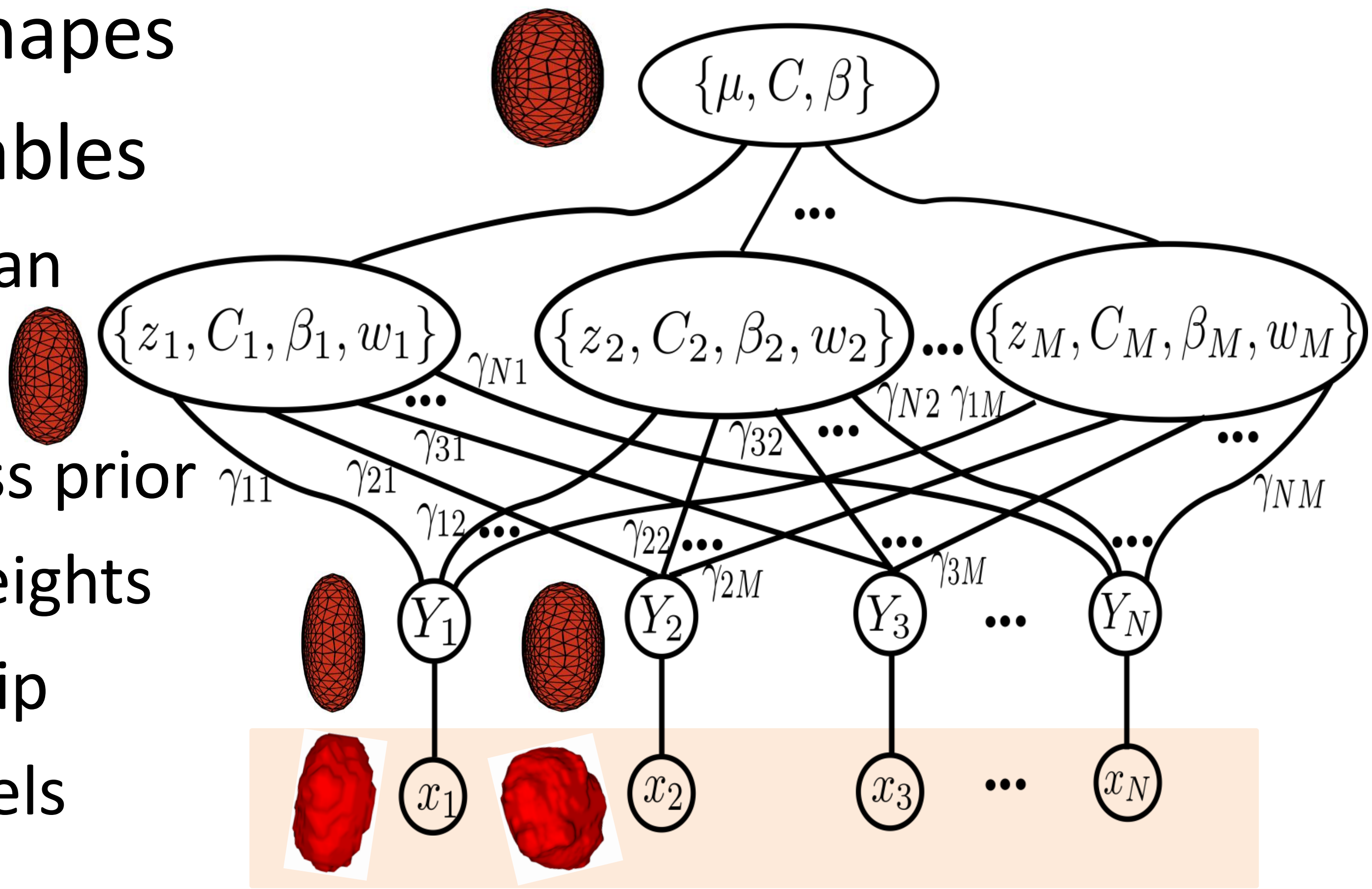

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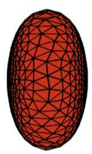
  - Covariance 
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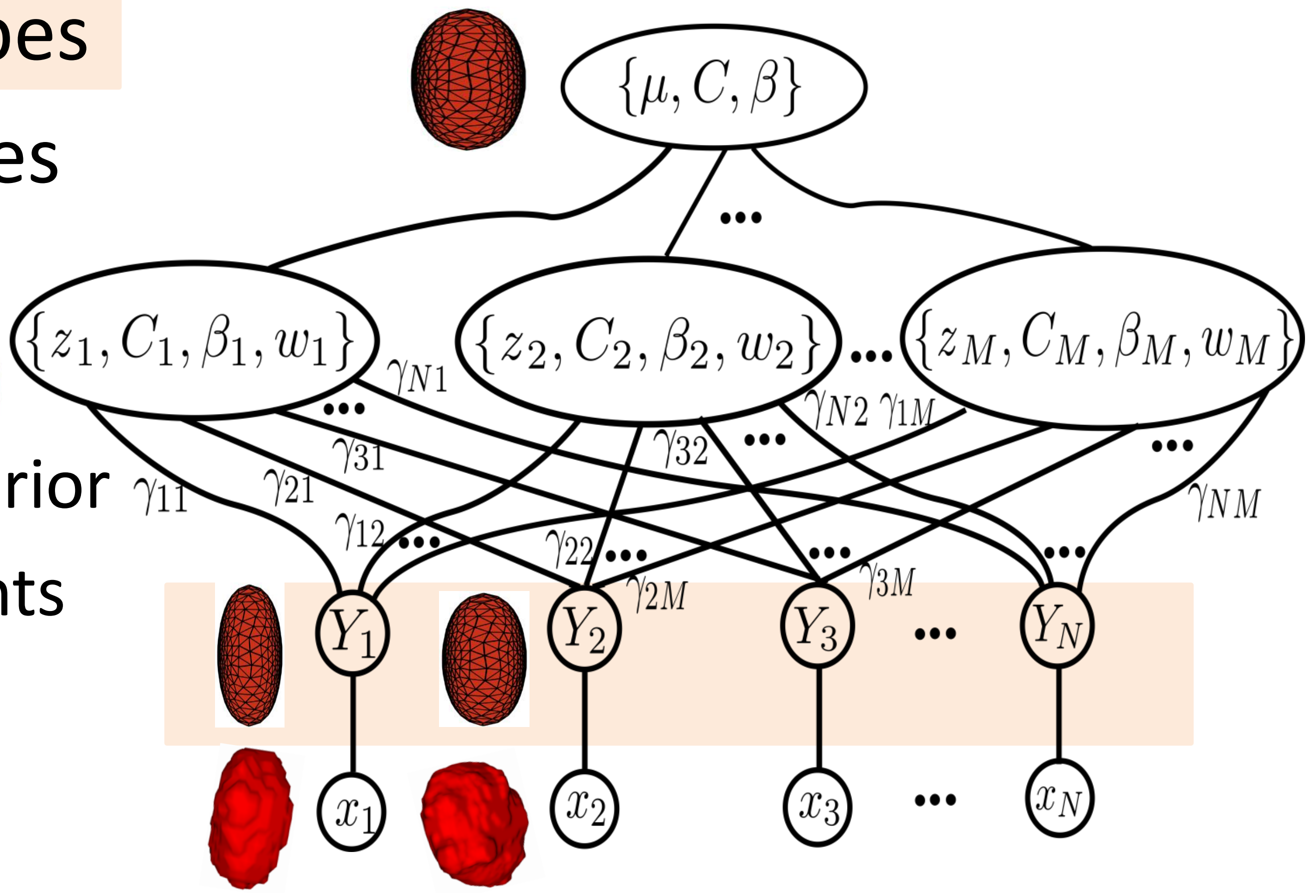

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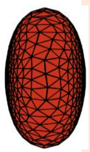
  - Covariance 
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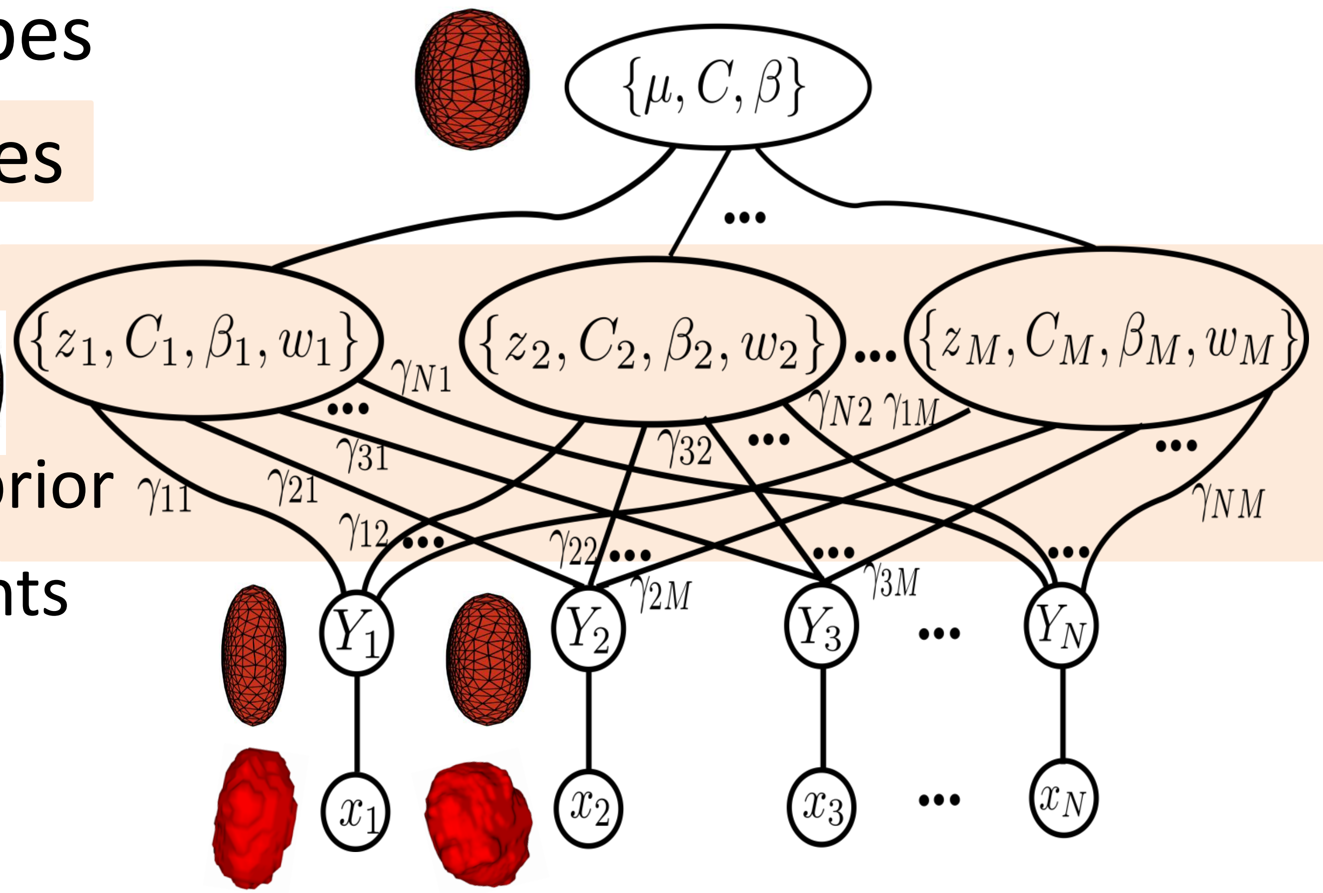

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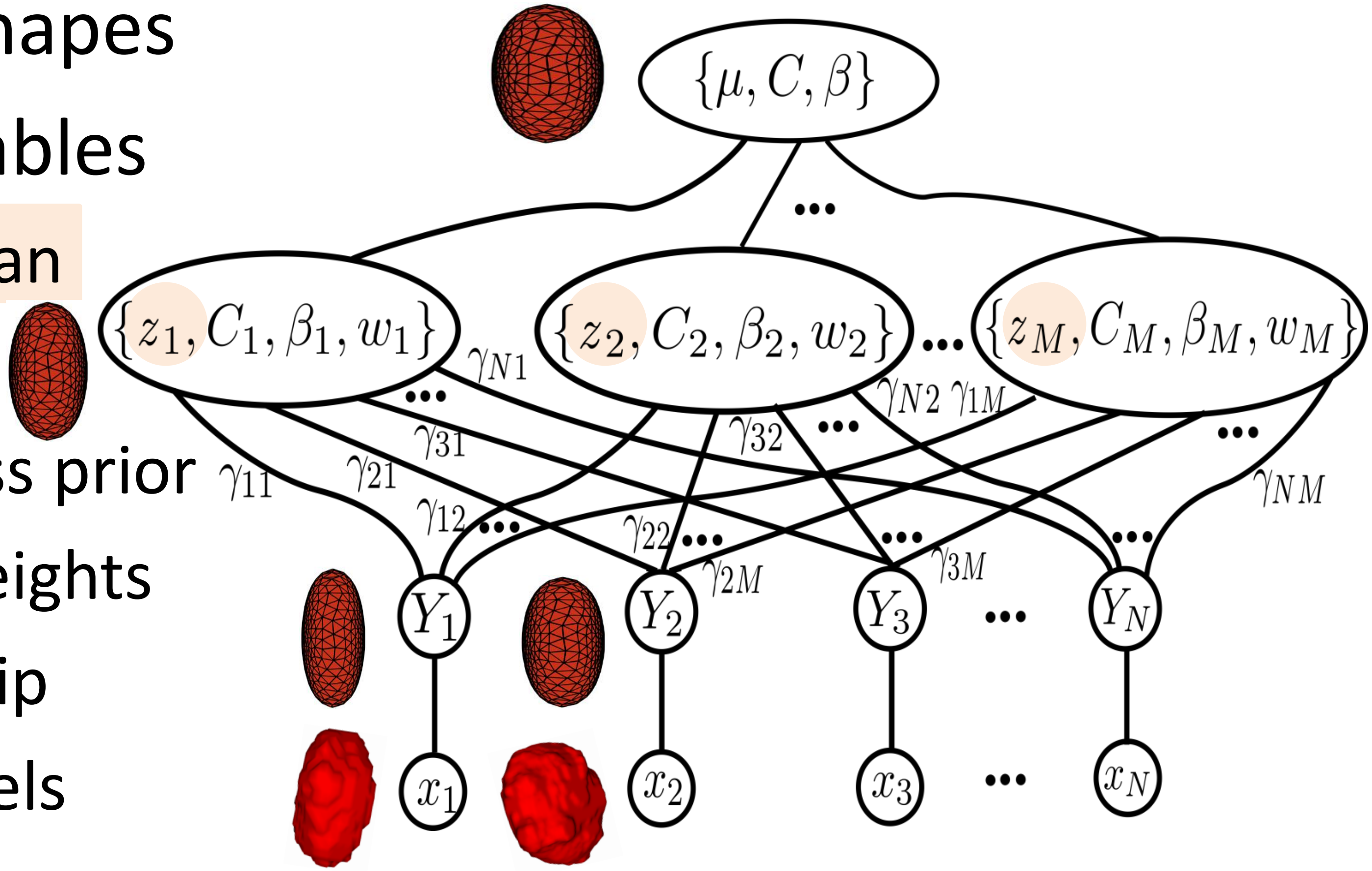

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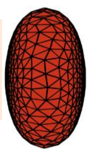
  - Covariance 
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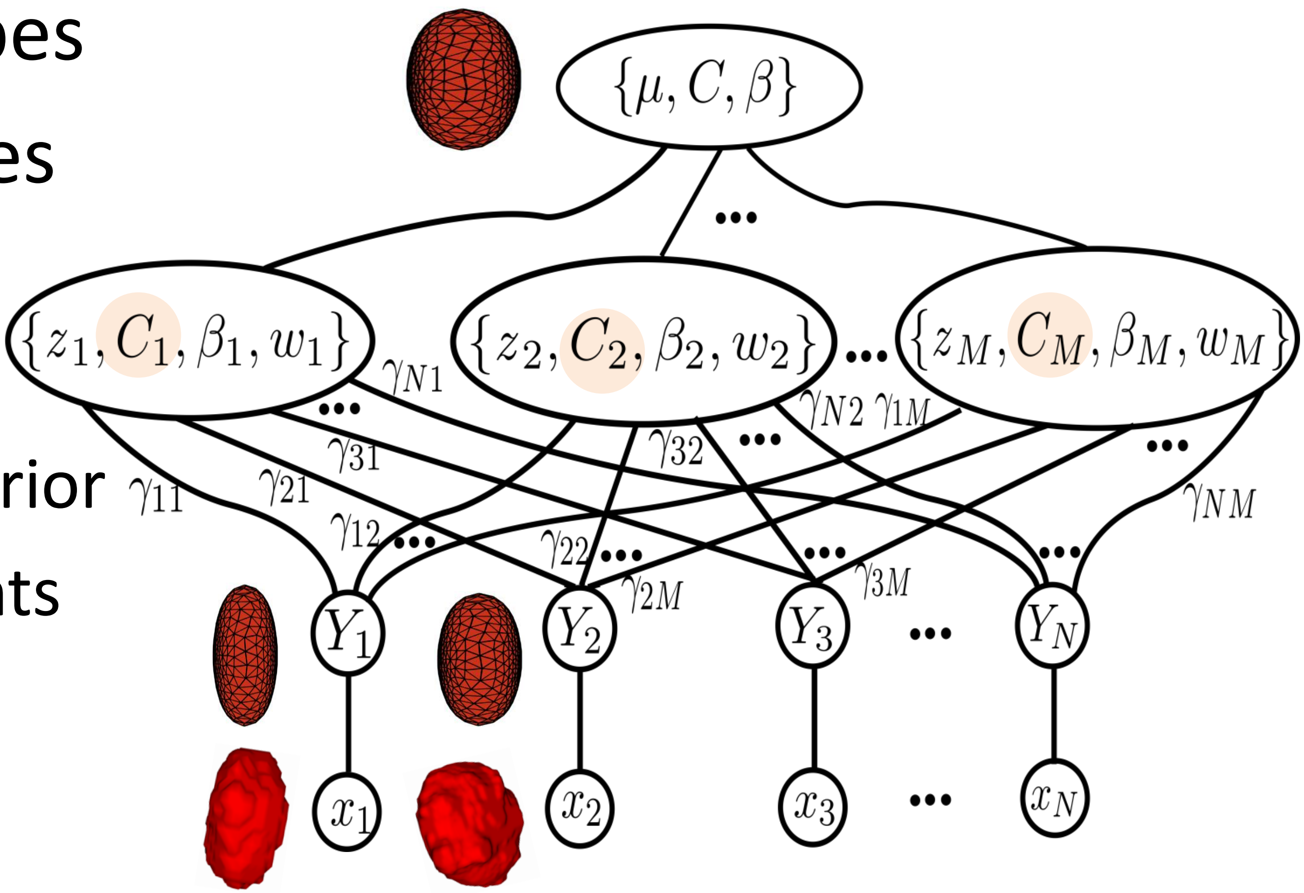

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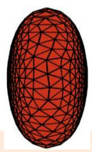
  - Covariance 
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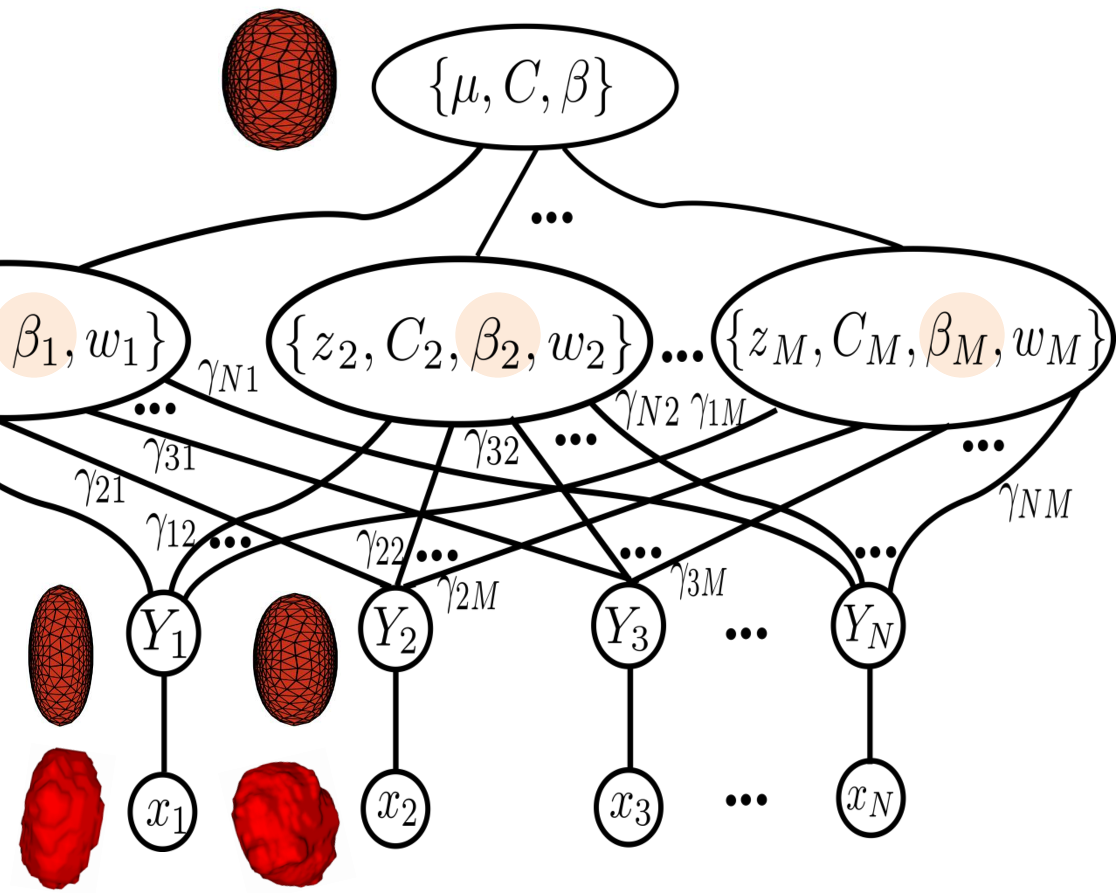

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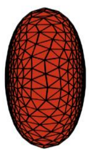
  - Covariance 
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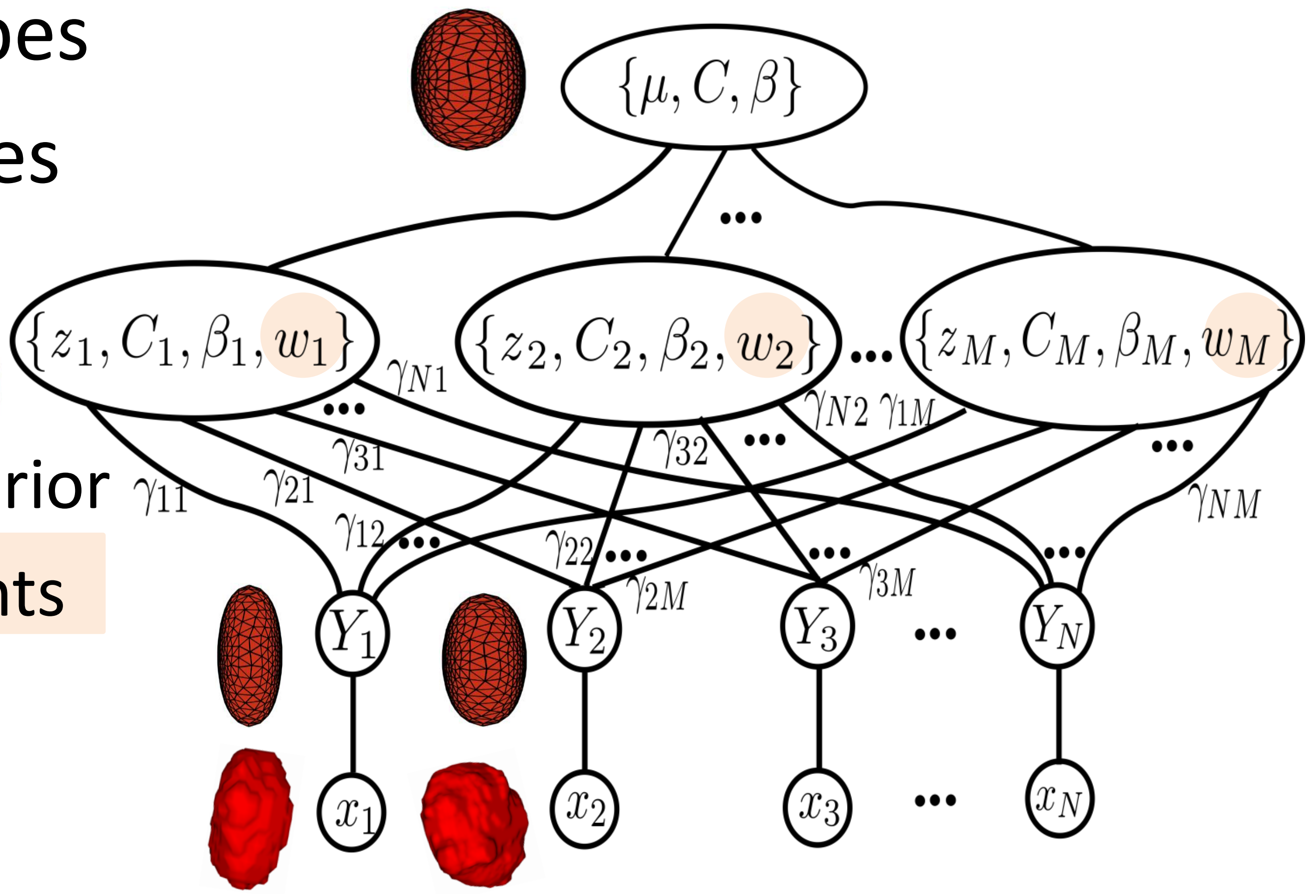

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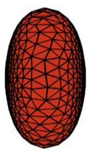
  - Covariance 
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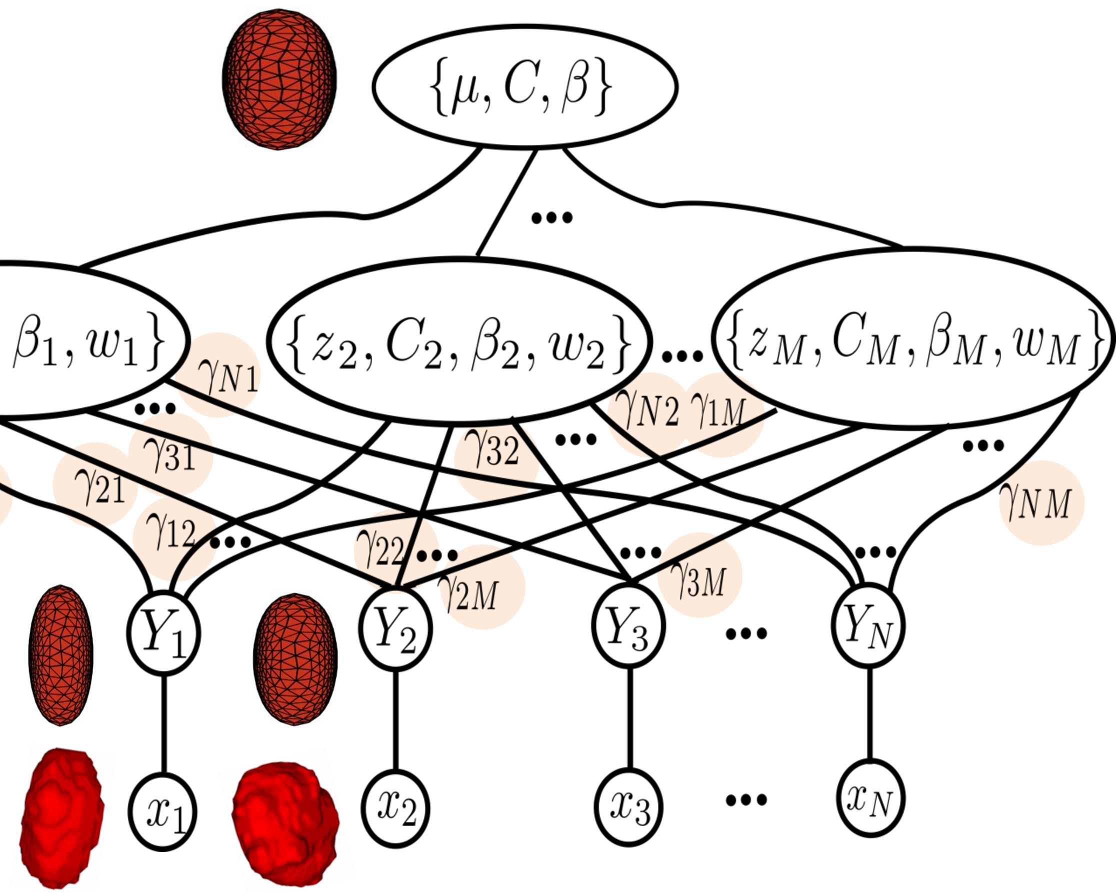
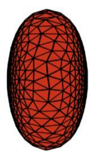
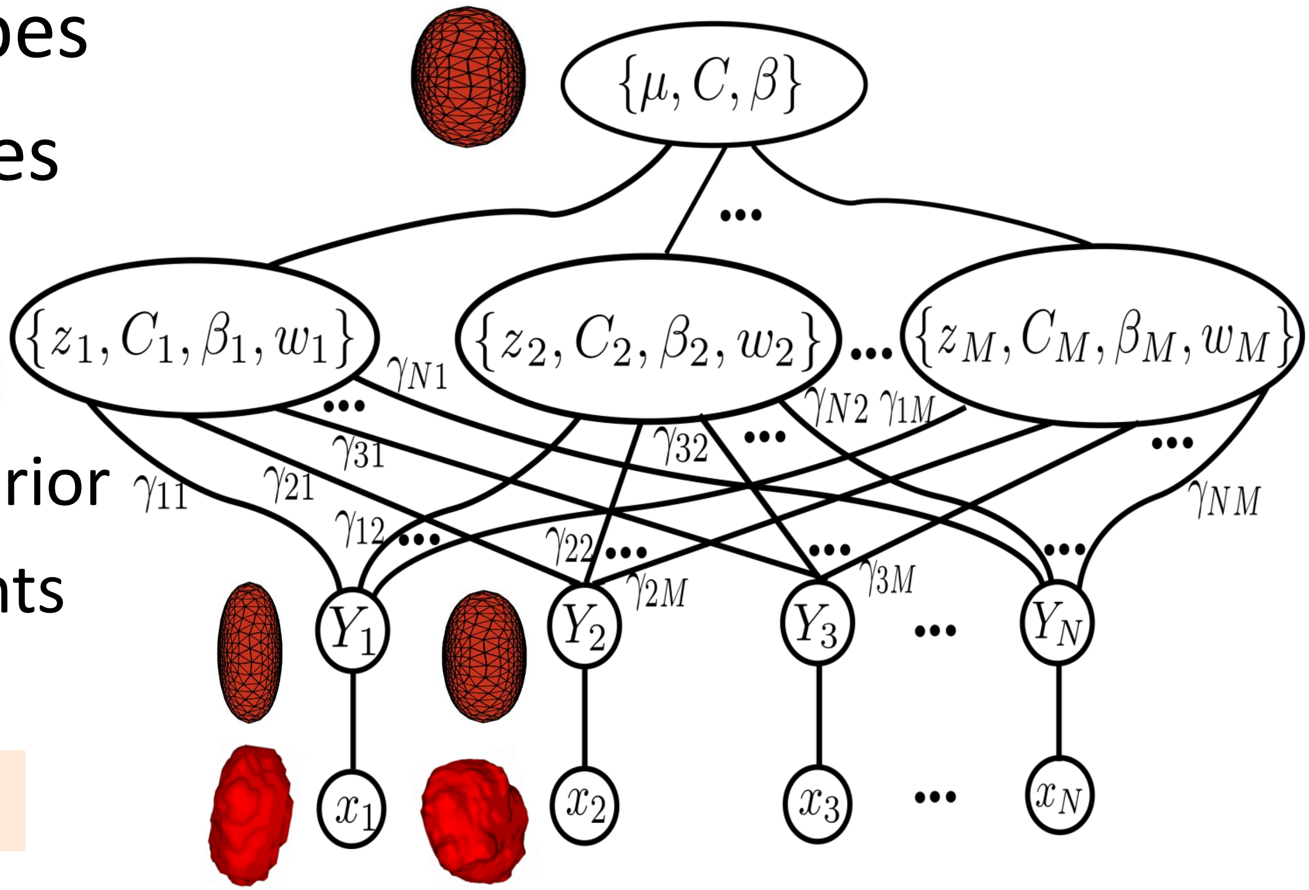


Modeling Clusters of Shapes

- Data
- Individual shapes
- Cluster variables
  - Cluster mean
  - Covariance 
  - Smoothness prior
  - Mixture weights
  - Membership
  - Cluster labels
- Population variables



$$\bar{\nu} := \{\nu_n \in \{1, 2, \cdots, M\}\}_{n=1}^{N}$$

Modeling Clusters of Shapes

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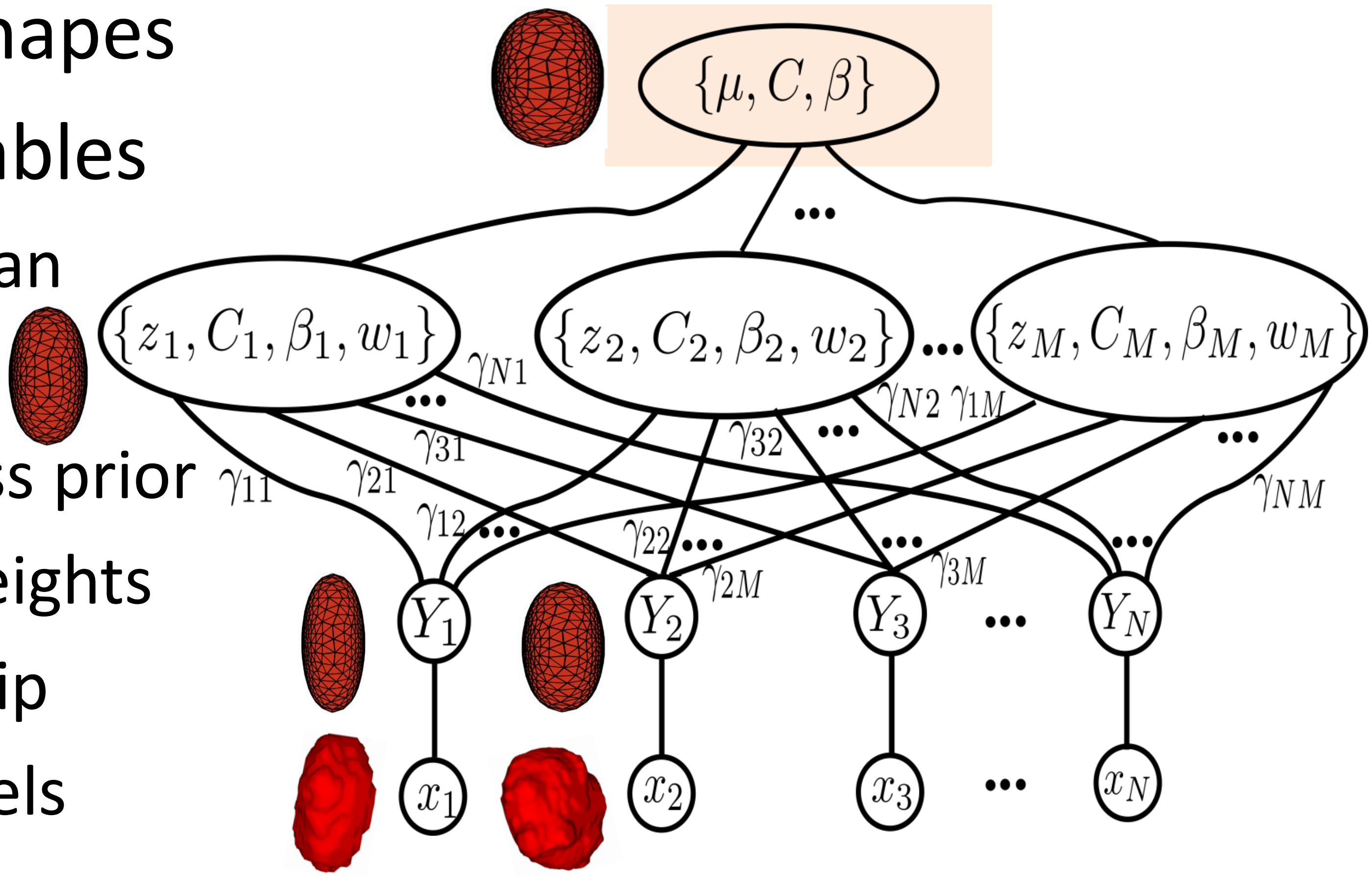

Modeling Clusters of Shapes

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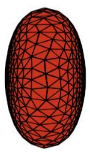
  - Covariance 
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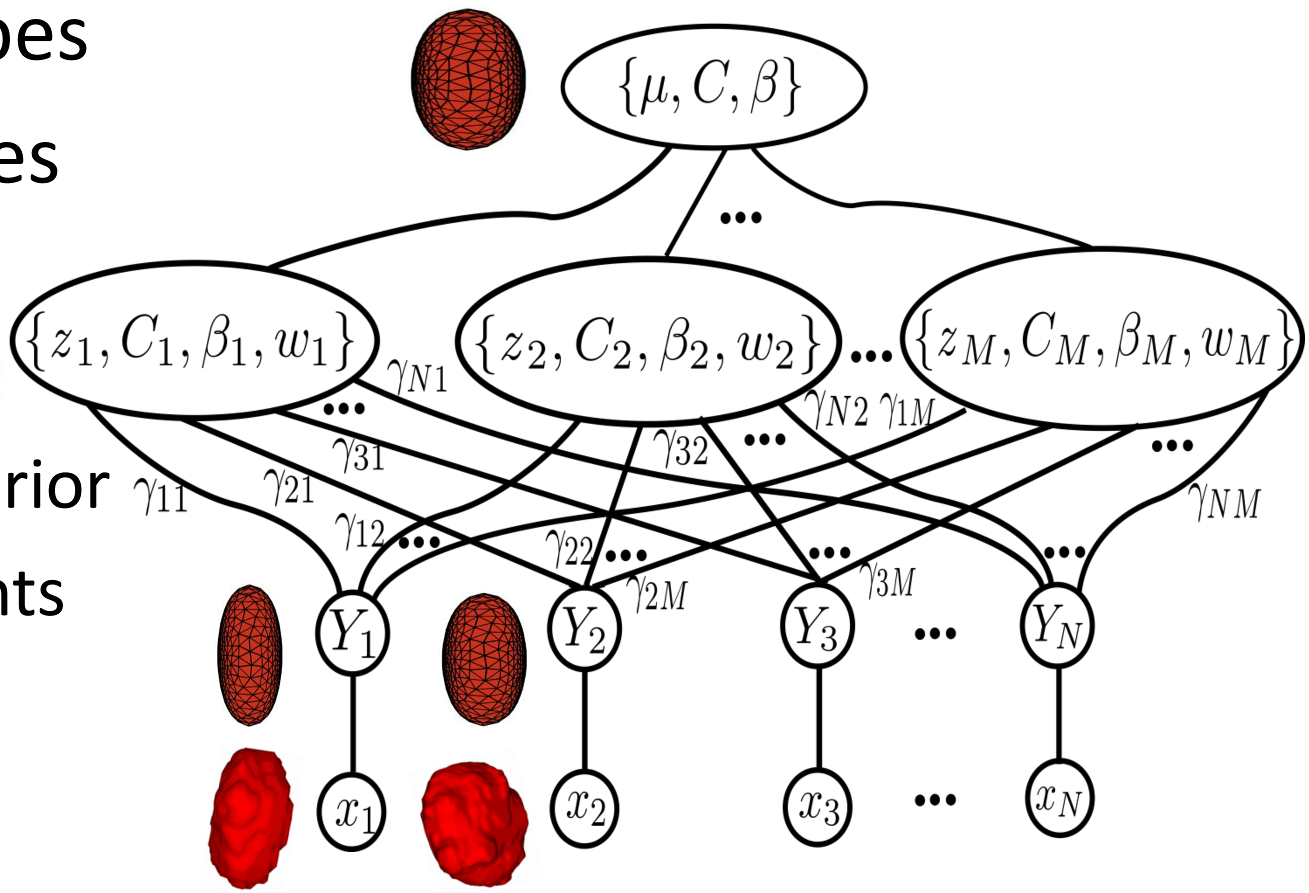
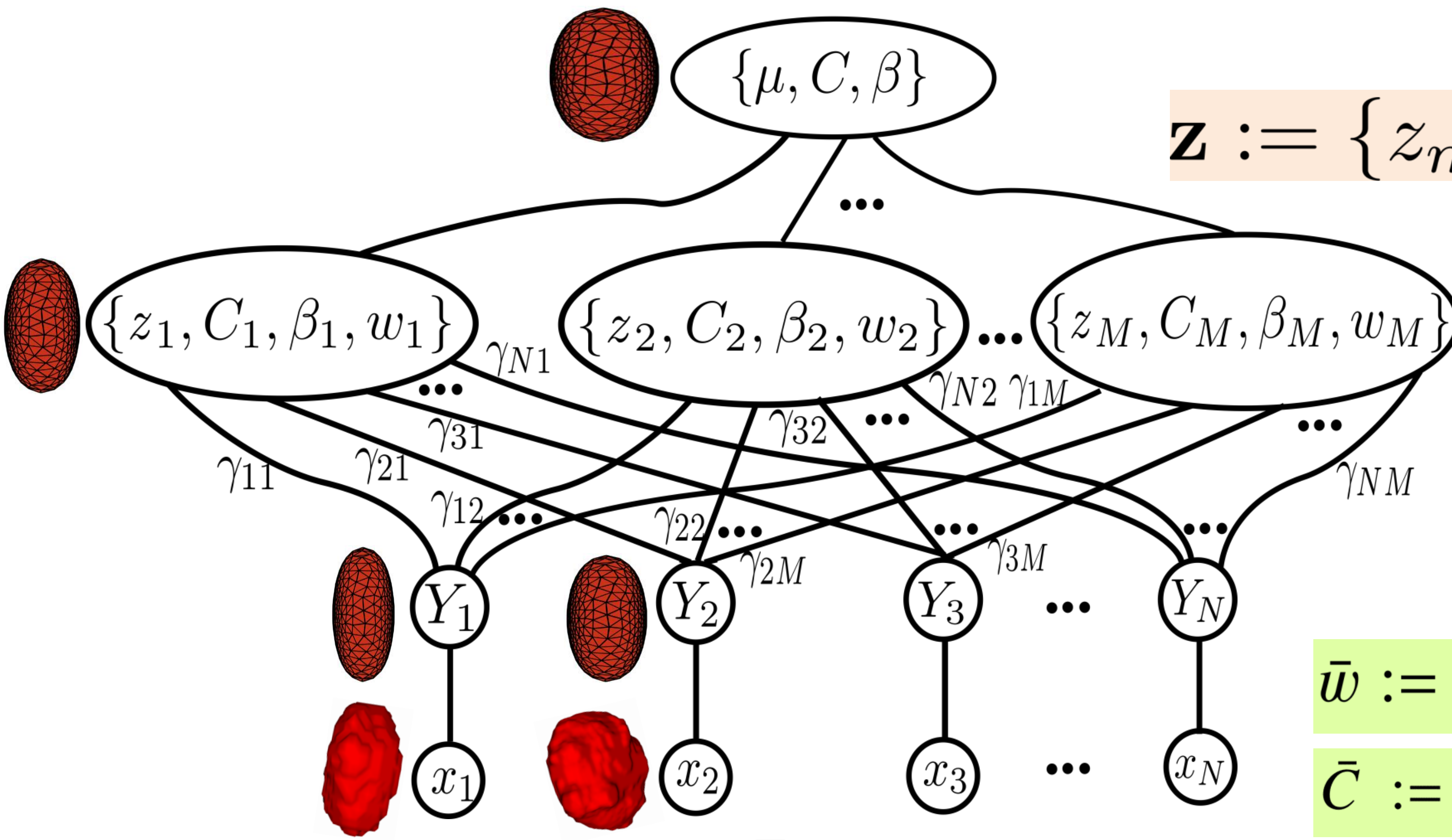


$$\bar{\nu} := \{\nu_n \in \{1, 2, \cdots, M\}\}_{n=1}^{N}$$

Hierarchical Shape Clustering



$\mathbf{z} := \{z_m\}_{m=1}^{M}$

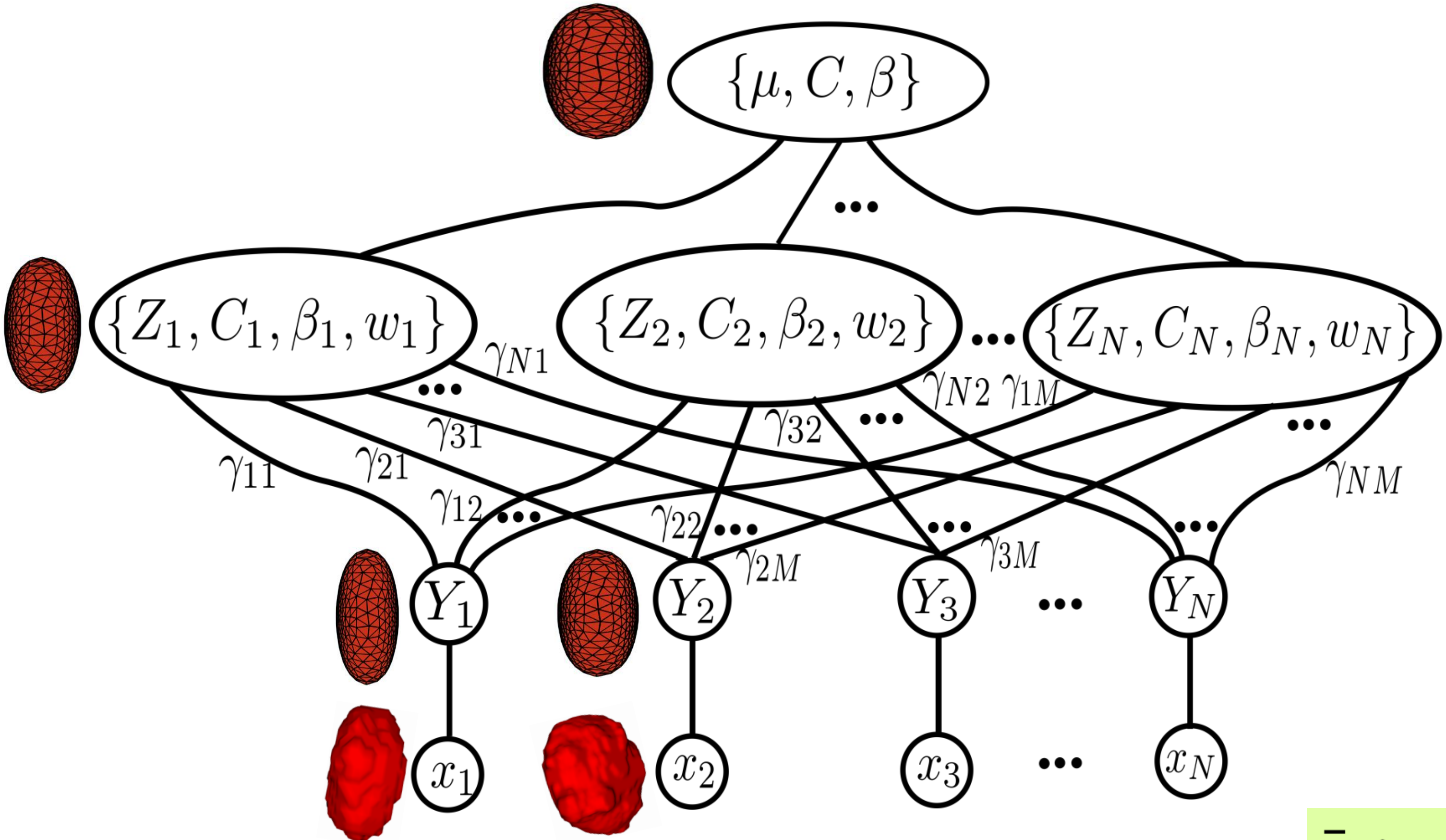
$\bar{w} := \{w_m\}_{m=1}^{M}$

$\bar{C} := \{C_m\}_{m=1}^{M}$

- With $\mathbf{z}$ and $\theta := \{\mu, C, \bar{C}, \bar{w}\}$ as parameters,

$$\max_{\mathbf{z},\theta} P(\mathbf{x}|\mathbf{z},\theta)P(\mathbf{z}|\theta) = \max_{\mathbf{z},\theta} \int P(\mathbf{x}, \mathbf{Y}, \bar{v}|\mathbf{z},\theta)P(\mathbf{z}|\theta)d\mathbf{Y}d\bar{v}$$

Hierarchical Shape Clustering



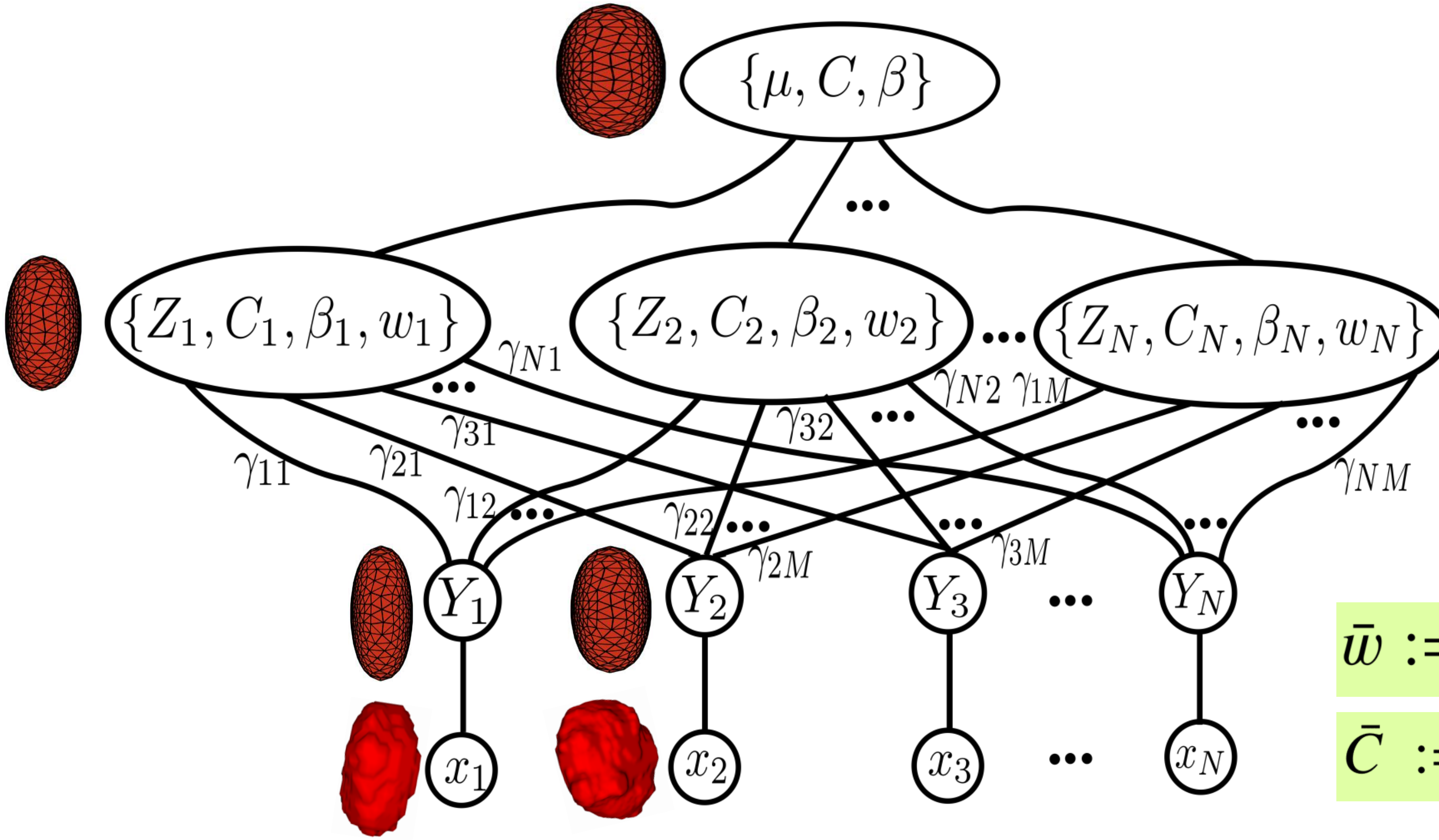
$\bar{w} := \{w_m\}_{m=1}^{M}$

$\bar{C} := \{C_m\}_{m=1}^{M}$

- With $\theta := \{\mu, C, \bar{C}, \bar{w}\}$ as parameter,

$$\prod_{n=1}^{N} \sum_{m=1}^{M} \int P(x_n|Y_n) P(Y_n|\nu_n = m, z_m, C_m) P(\nu_n = m|\theta) P(\mathbf{z}|\mu, C, \beta) dY_n$$

Hierarchical Shape Clustering



$\bar{w} := \{w_m\}_{m=1}^{M}$

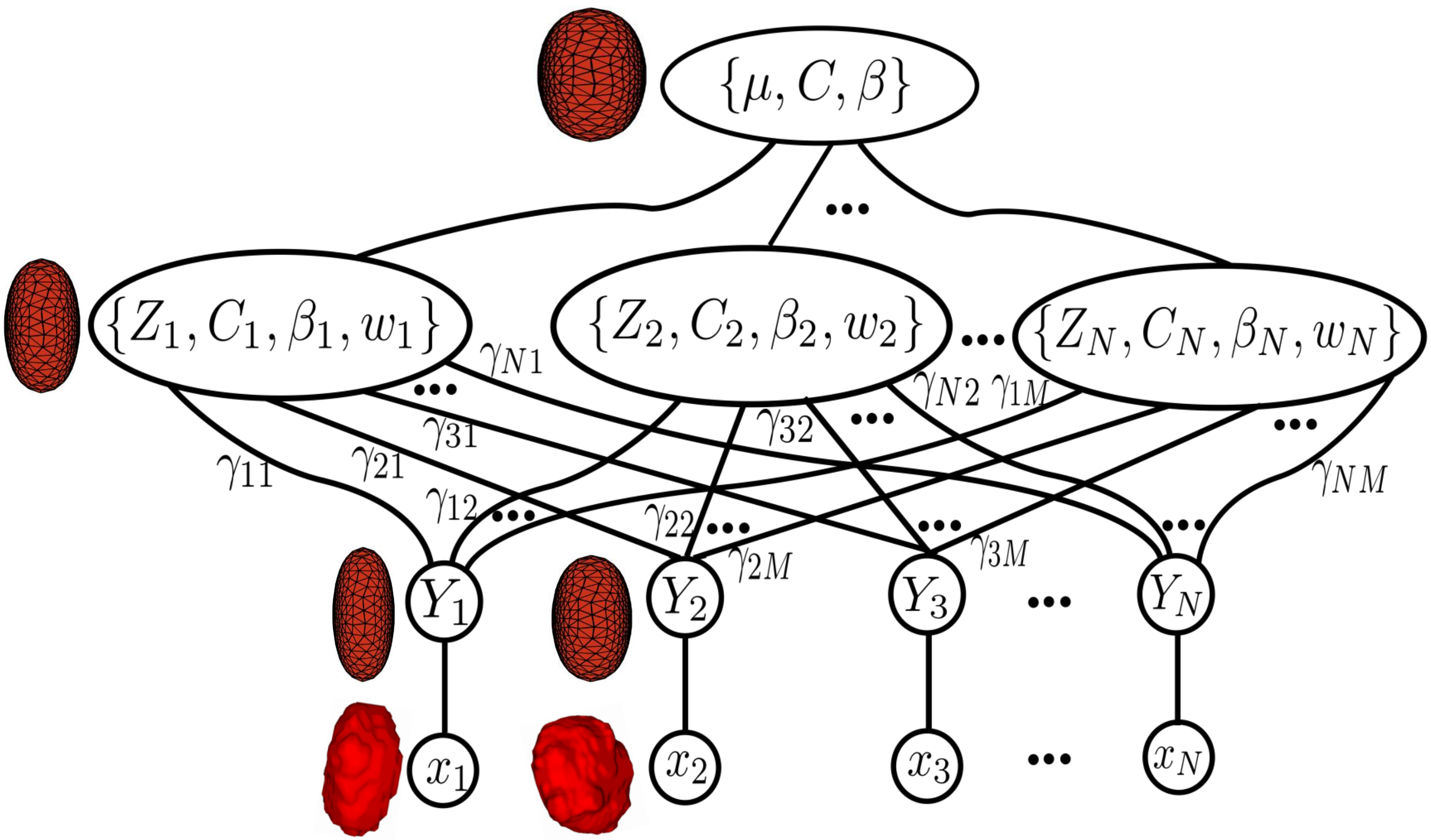
$\bar{C} := \{C_m\}_{m=1}^{M}$

- Optimize using MC-EM

$$\theta := \{\mu, C, \bar{C}, \bar{w}\}$$

$$\prod_{n=1}^{N} \sum_{m=1}^{M} \int P(x_n|Y_n) P(Y_n|\nu_n = m, z_m, C_m) P(\nu_n = m|\theta) P(\mathbf{z}|\mu, C, \beta) dY_n$$

Hierarchical Shape Clustering

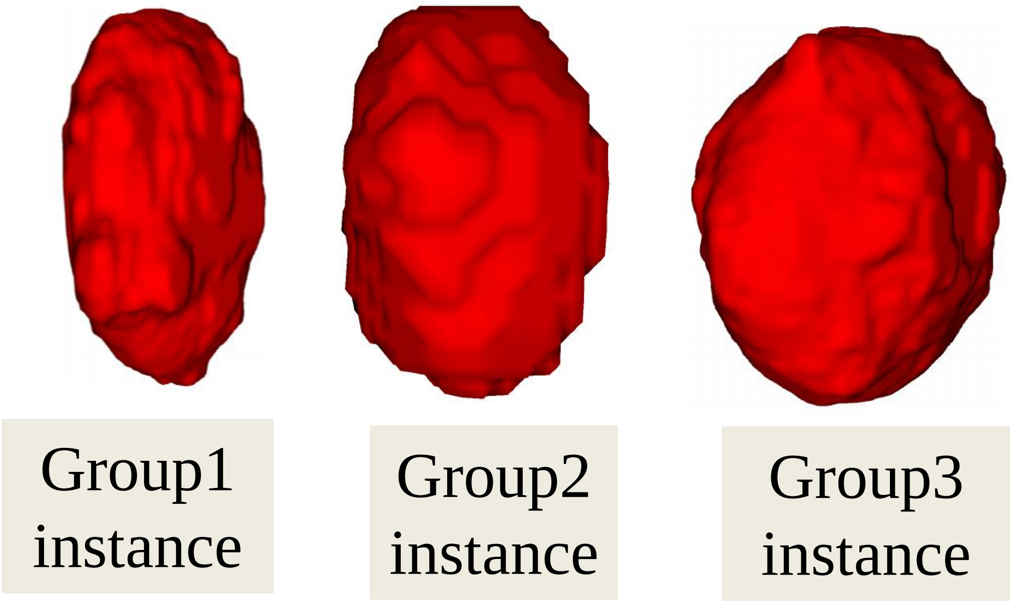


- Sampling using Leapfrog in Shape Space

$$\prod_{n=1}^{N} \sum_{m=1}^{M} \int P(x_n|Y_n) P(Y_n|\nu_n = m, z_m, C_m) \\ P(\nu_n = m|\theta) P(\mathbf{z}|\mu, C, \beta) dY_n$$

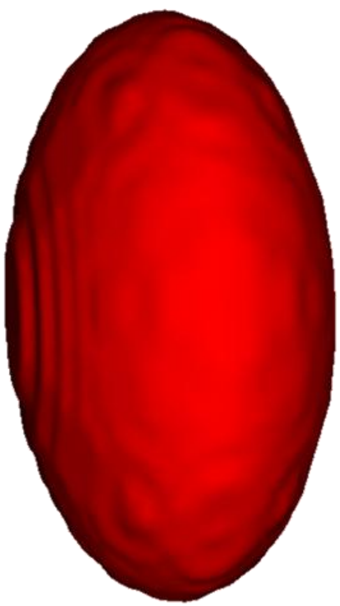
Clustering Evaluation Simulated Data

- Simulating data from 3 groups of 3D ellipsoids
  - Groups
    - 32 ellipsoids each
    - 2 axes lengths fixed to 30
    - 3rd axis length varies from 10 to 17 in group 1 16 to 24 in group 2 23 to 30 in group 3
    - Introduce random perturbations / bumps on surface
    - Evaluated clusters based on ground truth
    - Compared our results with **VBMixPCA** [*Gooya et.al., TPAMI 2018*]

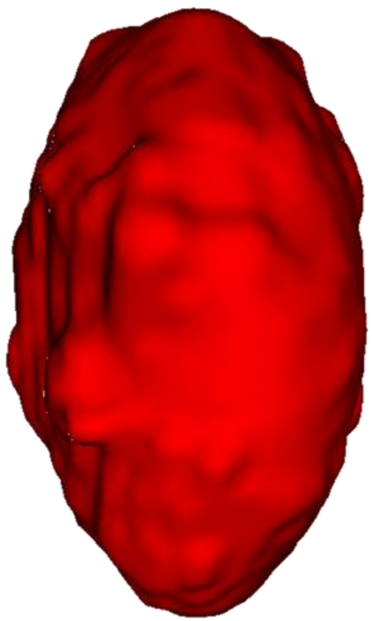


Clustering Evaluation Simulated Data

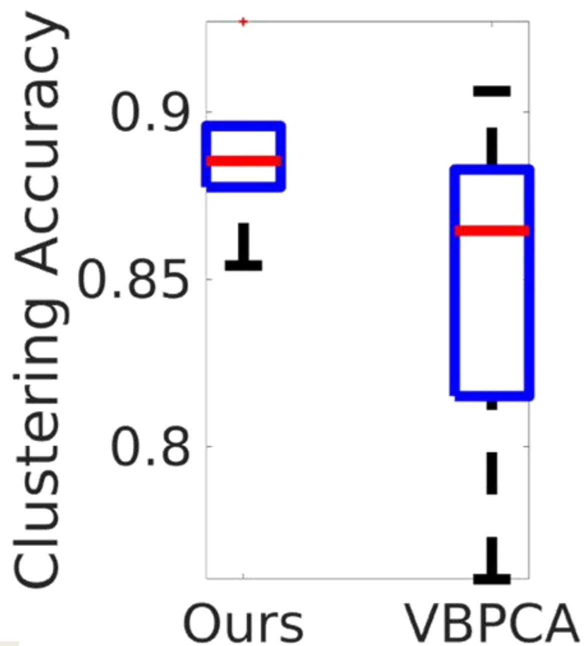
- Accuracy of clustering is calculated between true labels and estimated labels



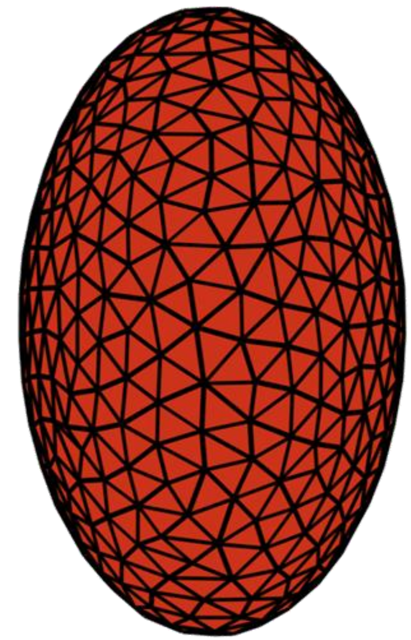
Ground truth



Corrupted mimicking human errors

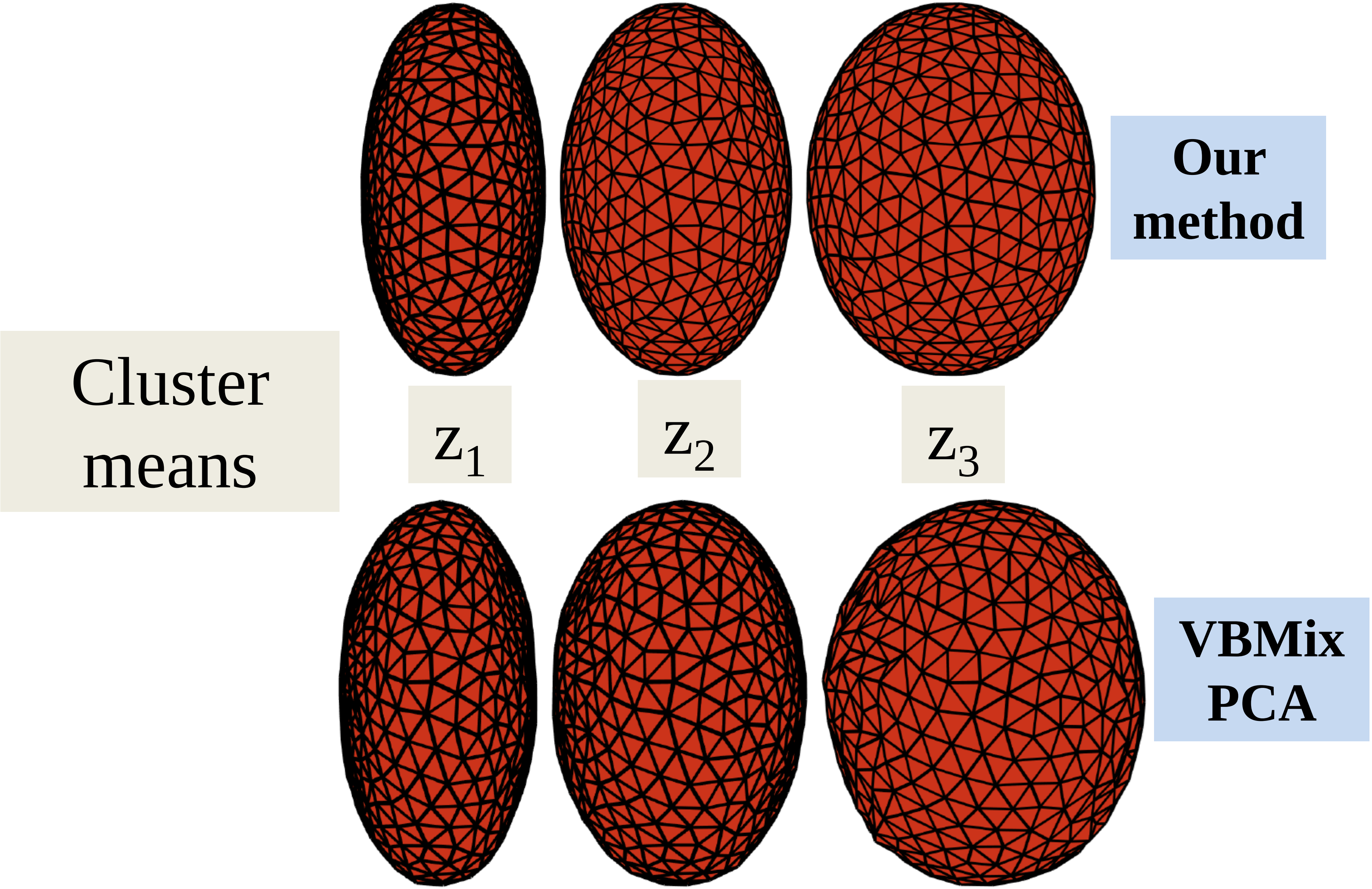


Boxplot of accuracy over 5 noisy instances



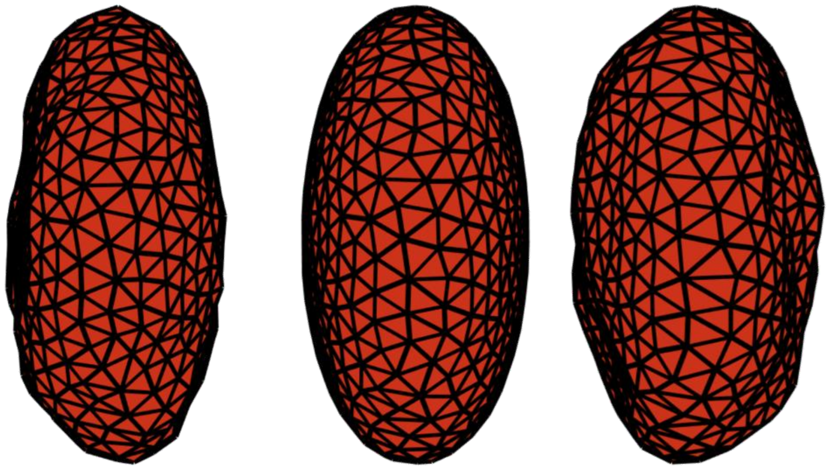
Population mean estimate of our method

Clustering Evaluation Simulated Data



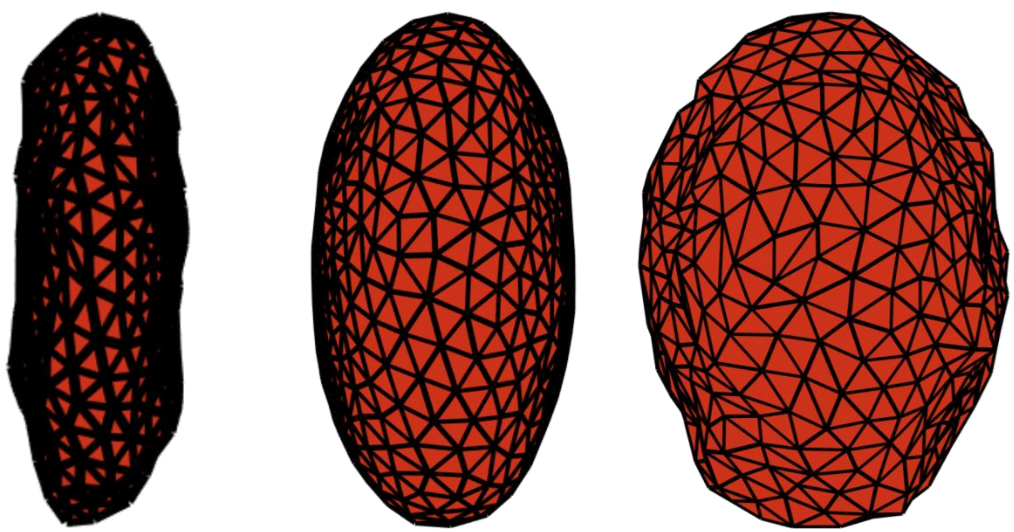
Clustering Evaluation Simulated Data

Principle mode of Variation around z_1



Our method

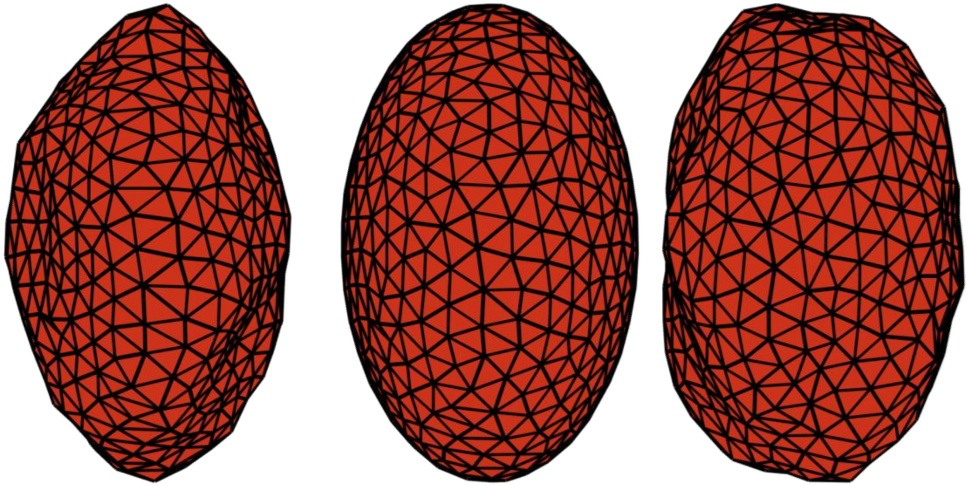
Cluster-1 mean z_1



VBMix PCA

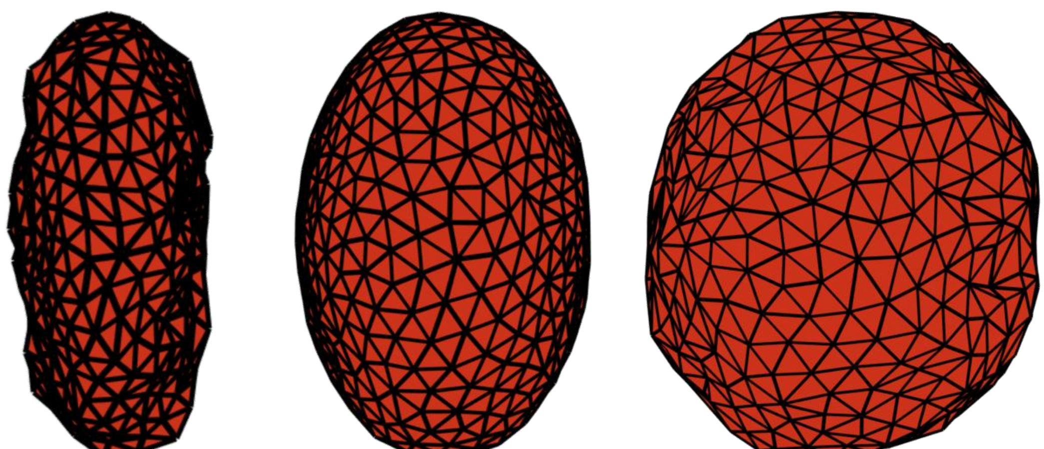
Clustering Evaluation Simulated Data

Principle mode of Variation around z_2



Our method

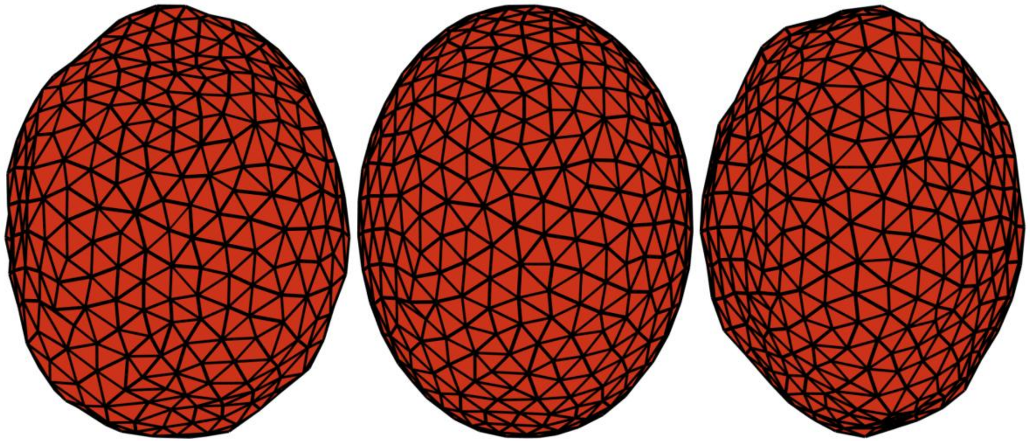
Cluster-2 mean z_2



VBMix PCA

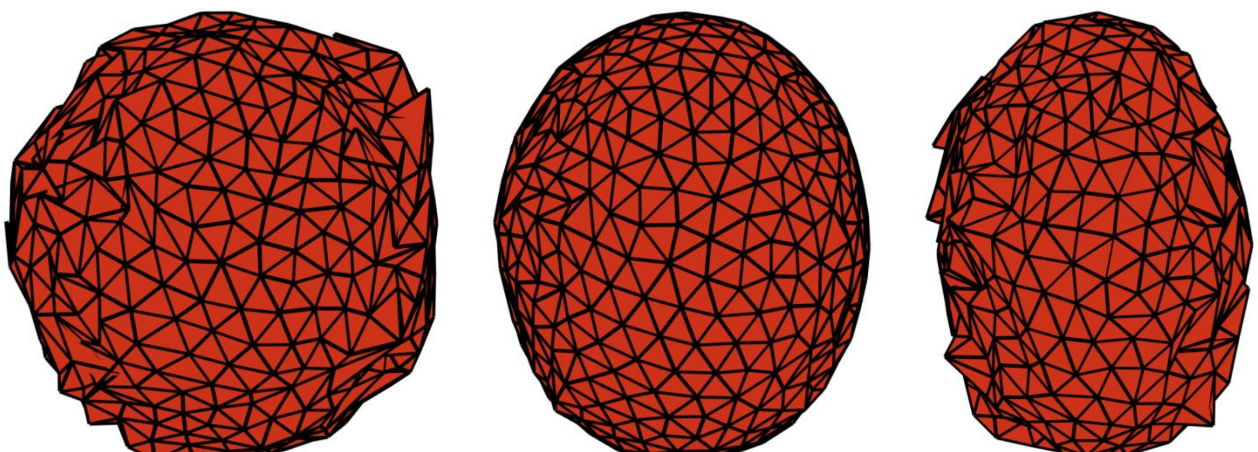
Clustering Evaluation Simulated Data

Principle mode of Variation around z_3



Our method

Cluster-3 mean z_3



VBMix PCA

Conclusion

- Proposed a novel hierarchical generative model for statistical shape analysis using point distribution model, in which pointsets lie in Kendall shape space
- Handles noisy segmentations
- Evaluated this framework for hypothesis testing
- Proposed Bayesian object segmentation using statistical shape prior, which extends deep neural nets and give improved segmentation
- Proposed hierarchical shape clustering framework based on Riemannian mixture component

Thank You